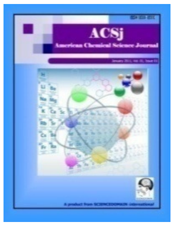

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Effect of Radiation on Unsteady MHD Flow of a Chemically Reacting Fluid Past a Hot Vertical Porous Plate: A Finite Difference Approach

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Authors' contributions

This whole work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

The paper deals with the effect of radiation on unsteady free convention flow of a viscous, incompressible, electrically conducting fluid past an infinite hot vertical porous plate embedded in porous medium. A magnetic field of uniform strength is applied normal to the fluid flow. Chemical reaction, viscous dissipation and Heat generation/absorption effects are included. Temperature of the plate is assumed to be span wise cosinusoidally fluctuating with time. In order to establish a mathematical convenience of converging the solution at a finite point ($\eta \to 1$), the governing equations of the problem are transformed to a new system of co-ordinates and then resulting equations are reduced to coupled non-linear ordinary differential equations of zeroth and first order, using appropriate perturbation technique. These zeroth and first order equations are solved numerically by making use of finite difference method and the simulation is carried out by coding in C-Program. Graphical results for velocity, temperature and concentration fields are presented and

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discussed at various parametric conditions. A research finding of this study, achieved that the temperature and velocity profiles are observed to be decreasing with the increasing values of radiation parameter.

Keywords: Radiation; magnetic field; chemical reaction; heat generation/absorption; finite difference method; span wise cosinusoidally fluctuating temperature.

NOMENCLATURE

ρ	*Density*
C_P	*Specific heat at constant pressure*
υ	*Kinematic viscosity*
k	*Thermal conductivity*
Gr	*Free convection parameter due to temperature*
Gm	*Free convection parameter due to concentration*
D	*Chemical molar diffusivity*
R	*Thermal radiation parameter*
M	*Magnetic parameter*
σ	*Electrical conductivity*
Q^*	*Heat generation/absorption coefficient*
Ec	*Eckert number*
β	*Coefficient of volume expansion for heat transfer.*
β^*	*Volumetric coefficient of expansion with species concentration*
K_1	*Rate of chemical reaction*
Sc	*Schmidt number*
T^*	*Temperature of the fluid*
Pr	*Prandtl number*
Q	*Heat generation/absorption parameter*
$\in$	*Small reference parameter*
a_R	*Rosseland radiation absorbtivity*
H_0	*Magnetic field*
C^*	*Species concentration*
K^*	*Permeability of the porous medium*
μ	*Viscosity of the fluid*
Ch	*Chemical reaction parameter*
K	*Permeability parameter*

1. INTRODUCTION

Several authors have dealt with heat flow and mass transfer over a vertical porous plate with variable suction, heat absorption/ generation, radiation and chemical reaction. Actually, many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment, Nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases one has to take into account the effects of radiation.

Several authors have considered the effect of radiation on Newtonian flows. Perdikis et al. [1] illustrated the heat transfer of a micro polar fluid in the presence of radiation. Abdus Sattar and Hamid Kalim [2] investigated the unsteady free convection interaction with thermal radiation in the boundary layer flow past a vertical porous plate. Raptis [3] studied the effect of radiation on the flow of a micro-polar fluid past a continuous moving plate. Raptis et al. [4] studied the viscoelastic flow by the presence of radiation. Raptis [5] discussed the effects of radiation on steady flow of a viscous fluid through a porous medium bounded by a porous plate subjected to a constant suction velocity. Elbashbeshby and

Bazid [6] have reported the effect of radiation on forced convection flow of a micro polar fluid over a horizontal plate. Chamkha et al. [7] analyzed the effect of radiation on free convection flow past a semi infinite vertical plate with mass transfer. Ganesan and Loganathan [8] studied the radiation and Mass transfer effects on flow of a viscous incompressible fluid past a moving cylinder. Kim et al. [9] analysed the effect of radiation on transient mixed convection flow of a micropolar fluid past a moving semi infinite vertical porous plate. Makinde [10] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid. Ramachandra Prasad et al. [11] considered the effects radiation and mass transfer on two dimensional flow past an infinite vertical plate.

Moreover, when the radiative heat transfer takes place, the fluid involved can be electrically conducting in the sense that it is ionized owing to high operating temperature. In such case one cannot neglect the effect of magnetic field, which is one of the important parameters by which the cooling rate can be controlled in nuclear reactors and the product of the desired quality can be achieved. So in the recent years the subject of Magneto hydrodynamics has attracted the attention of many researchers due to the increasing number of various technical applications to problems of geophysical and astrophysical significance. MHD convection plays predominant role in various industrial applications. eg:- Include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials, MHD power generators etc. In the field of power generation, MHD receiving considerable attention due to possibilities, it offers for much higher thermal efficiencies in power plants. The use of MHD flow meters is also prominent in medicine, when the flow rate of blood for saline solutions measured. Due to its increasing number of various technical applications, several authors [12-14] have dealt the problems related to magneto hydrodynamic flows. Takhar et al. [15] reported the effect of magnetic field on free-convection flow of a radiation gas past a semi infinite vertical plate. Raptis and Massalas [16] studied the magneto-hydrodynamic flow past a plate by the presence of radiation. Sharma et al. [17] discussed the effect of radiation on magneto-hydrodynamic fluctuating free convective flow with embedded in porous medium having variable permeability and heat Source/Sink. Sharma et al. [18] have reported on the radiation effect with simultaneous thermal and mass diffusion in hydro-magnetic mixed convection flow from a vertical surface with ohmic heating. Perdikis and Rapti [19] presented an analysis of unsteady hydro magnetic flow in the presence of radiation. Chaudhary and Preethi Jain [20] presented an analysis to study the effect of radiation on the hydro-magnetic free convection flow of an electrically conducting micro polar fluid past a vertical porous plate through a porous medium in slip-flow regime.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have received a considerable amount of attention in recent years. In processes such as evaporation at the surface of water body, drying, energy transfer in wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Chemical reaction can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. If the rate of reaction is directly proportional to the concentration itself a reaction is said to be first order, which has many applications in different chemical engineering processes and other industrial applications such as polymer production, manufacturing of ceramics or glassware and food processing [21]. Das et al. [22] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan [23] and Muthucumarswamy [24] studied first order homogeneous chemical reaction on flow past infinite vertical plate. Periasamy et al. [25] discussed heat and mass transfer effect along a wedge with heat source and concentration in the presence of suction/injection taking into account the chemical reaction of first order. Prakash and Ogulu [26] have studied the effect of thermal radiation, time-dependent suction and chemical reaction on two-dimensional flow of an incompressible Boussinesq fluid. Ibrahim et al. [27] analysed the effects of the chemical reaction and radiation absorption on transient hydro-magnetic free-convention flow past a semi infinite vertical permeable moving plate with wall transpiration and heat source. Sudheer Babu and Satyanarayana [28] discussed the effects of the chemical reaction and radiation absorption in the presence of magnetic field on free convection flow through porous medium with variable suction. Dulal Pal et al. [29] has made the Perturbation analysis to study the effects thermal radiation and chemical reaction on magneto-hydrodynamic unsteady heat and mass transfer in a boundary layer flow past a vertical permeable plate in the slip flow regime.

To consider the situation of occurring chemical reaction, we include the contribution of internal heat generation/absorption, which plays a vital role in maintaining heat transfer at desired level in the applications of Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles. Due to the practical application of the heat generation or absorption effect, several authors have addressed this issue on different geometry considering various flow conditions (see for examples [30–36]).

In the above stated studies, most of the investigators have restricted themselves to two-dimensional flows only. However, there may arise situations where the flow field may be essentially three-dimensional, for example, when variation of the permeability distribution is transverse to the potential flow. The effect of such a transverse permeability distribution of the porous medium bounded by horizontal flat plate has been studied by Sing and Verma [37] and Singh et al. [38]. In addition to this, Singh and Sharma [39] studied the three-dimensional free-convective flow and heat transfer through a porous medium with periodic permeability. Later this same study with mass transfer was extended by Varshney and Singh [40]. Jain et al. [41] analysed the effects of periodic temperature and periodic permeability on three-dimensional free convective flow through porous medium in slip flow regime. Srihari and Anand Rao [42] analysed the effect of magnetic field on three-dimensional free-convective heat and mass transfer flow through a porous medium with periodic permeability. Besides, Ahmed and Talukdar [43] discussed the effect of chemical reaction on an oscillatory three-dimensional flow with mass transfer past a vertical plate with thermal diffusion in the presence of heat sink. Recently, Singh and Rakesh Kumar [44] studied the effect of chemical reaction on unsteady magneto-hydrodynamic free convention heat and mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite hot vertical porous plate with heat generation/absorption through porous medium, when the plate temperature is span wise cosinusoidally floating with time. More recently, Hayat et al. [45] investigated the three-dimensional flow of viscous fluid with convective boundary conditions and heat generation/ absorption.

In most of the earlier studies, a series expansion method was adopted to obtain the solution of three dimensional flows and in all these attempts the effect of radiation is of no significant consideration on the flow field. Usually obtaining exact solution for these type of three-dimensional flows is very difficult because of its highly non-linearity, however, in the present paper a numerical attempt is made to study the effect of radiation on unsteady free convention flow of a viscous, incompressible, electrically conducting fluid past an infinite vertical porous plate embedded in porous medium with heat generation/absorption. Chemical reaction and viscous dissipation effects are included. A magnetic field of uniform strength is applied normal to the fluid flow. It is an extension of the work of Singh and Rakesh Kumar [44] with radiation.

2. MATHEMATICAL FORMULATION

We consider the flow past an infinite, hot porous plate being vertically on x^*-z^* plane. The x^* axis is oriented in the direction of the buoyancy force and y^* axis is taken perpendicular to the plane of the plate. A magnetic field of uniform strength Ho is introduced normal to the plane of the plate. Let (u^*, v^*, w^*) be the components of velocity in the (x^*, y^*, z^*) directions respectively. The plate is being considered infinite in x^*-directions; hence all physical quantities will be independent of x^*. Since the plate is subjected to a constant suction velocity, i.e. $v^* = -V$, [29], w^* is independent of z^* and therefore throughout the problem $w^*=0$ is assumed. The temperature of the plate is considered to vary span wise cosinusoidally fluctuating with time and assumed to be of the form.

$$T_w^*(z^*,t^*) = T_{0^*}^* + \varepsilon(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{l} - \omega^* t^*\right)$$

Where T_0^*, T_∞^* and T_w^* are the mean, ambient temperature and wall temperature of the plate respectively, ω^* is the frequency, t^* is the time,

l is the wave length and ε is a small parameter i.e, $\varepsilon << 1$.

We further assume that (i) the magnetic Reynolds number is small so that the induced magnetic field is negligible in comparison to the applied magnetic field (ii) the effects of Joule heating is negligible as small velocity is usually encountered in the free convection flows, (iii) no external electric field is applied and effect of polarization of ionized fluid is negligible. Therefore, electric field is assumed to be zero, (iv) there exists a first order chemical reaction between the fluid and species concentration, (v) the level of species concentration is very low so that the heat generated during chemical reaction can be neglected.

Diagram 2.1 shows the geometry of 3-D flow past an infinite, hot porous plate which is placed vertically in the x^*-z^* plane. The x* axis is taken in the direction of the buoyancy force and y^* axis is taken normal to the plane of the plate. A transverse magnetic field is applied on the fluid flow. (u^*, v^*, w^*) be the components of velocity in the (x*, y*, z*) directions respectively. The assumption considered in the modeling is to be an infinite plate in the x*-directions.

Using the Boussinesq and boundary layer approximation, the governing equations for this problem can be written as follows:

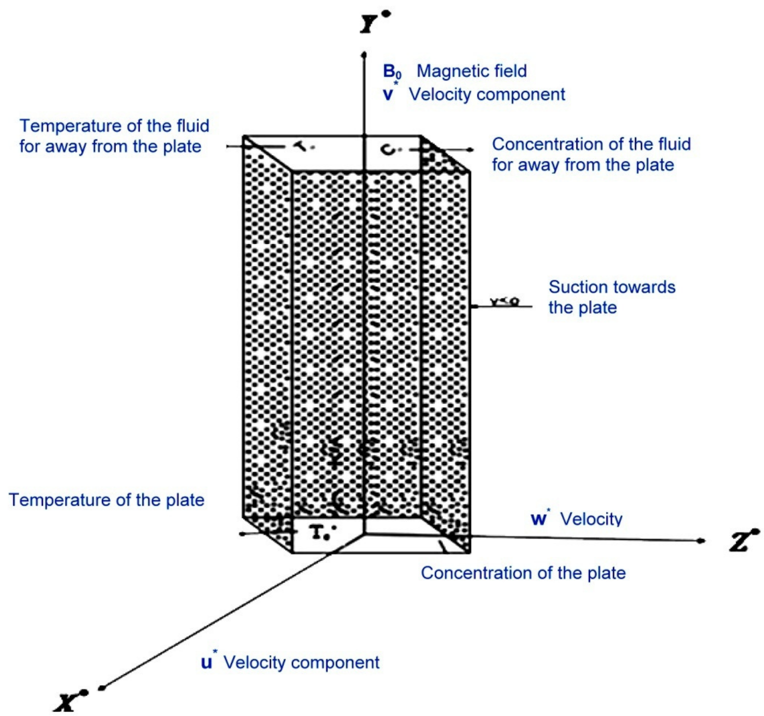

Diagram 2.1. Schematic of porous medium at different velocity components

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -V, V > 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \upsilon\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + g\beta(T^* - T^*_\infty) + g\beta_c(c^* - c^*_\infty) - \frac{\sigma B_0^2}{\rho}u^* - \frac{\upsilon}{K^*}u^* \tag{2}$$

$$\left(\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \frac{k}{\rho C_P}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \frac{\mu}{\rho C_P}\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right\} - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y^*} + \frac{Q^*(T^* - T^*_\infty)}{\rho C_P} \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\left(\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}\right) - K_1(c^* - c^*_\infty) \tag{4}$$

The radiative heat flux terms by using Rosseland approximation [19,46] are given by

$$q_r = \frac{-4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y^*} \tag{5}$$

The boundary conditions of the problem are

$$u^* = 0,\ T^* = T_0^* + \in (T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{l} - \omega^* t^*\right),\ C^* = C_0^* \quad \text{at } y = 0 \tag{6}$$

$$u^* = 0,\ T^* = T_\infty^*,\ C^* = C_\infty^* \quad \text{as } y \to \infty$$

We assume that the temperature differences within the flow are such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ as follows

$$f(T) = f(T_\infty) + (T - T_\infty)f'(T_\infty) + \frac{(T-T_\infty)^2}{2!}f''(T_\infty) + \ldots, \tag{7}$$

Where $f(T) = T^4$, then $f'(T) = 4T^3$, $f''(T) = 12T^2$

Simplifying (7), we get,

$$T^4 = T_\infty^4 + 4(T - T_\infty)T_\infty^3 + 12\frac{(T-T_\infty)^2}{2!}T_\infty^2 + \ldots$$

In the above **Taylors** expansion, neglecting the higher order terms, we have [19]

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using (8) in (5) and then (5) in (3), equation of energy (3) gives

$$\left(\frac{\partial T^*}{\partial t^*} + V^*\frac{\partial T^*}{\partial y^*}\right) = \frac{k}{\rho C_P}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \frac{\mu}{\rho C_P}\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right\} + \frac{16\sigma^* T_\alpha^3}{3\rho C_P a_R}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Q^*(T^* - T_\infty^*)}{\rho C_P} \tag{9}$$

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{l},\ z = \frac{z^*}{l},\ u = \frac{u^*}{V},\ \theta = \frac{T^* - T_\infty^*}{T_0^* - T_\infty^*},\ \emptyset = \frac{C^* - C_\infty^*}{C_0^* - C_\infty^*}$$

$$t = \omega^* t^*,\ Ec = \frac{V^2}{C_P(T_0^* - T_\infty^*)},\ Pr = \frac{\mu C_P}{k}, Re = \frac{Vl}{\upsilon},\ Gr = \frac{\upsilon g\beta(T_0^* - T_\infty^*)}{V^3} \tag{10}$$

$$Gm = \frac{\upsilon g\beta_c(c_0^* - c_\infty^*)}{V^3}, \omega = \frac{\omega^* l^2}{\upsilon}, Sc = \frac{\upsilon}{D}, M^2 = \frac{\sigma B_0^2 l^2}{\mu}$$

$$R = \frac{k a_R}{4\sigma^* T_\infty^3}, Ch = \frac{l^2 k_1}{\upsilon}, Q = \frac{Q^* l^2}{\upsilon\rho c_p}, K = \frac{K^*}{l^2},$$

in to the equations (1),(2),(4) and (9) the following equations are obtained in dimension less form:

$$\omega\frac{\partial u}{\partial t} - Re\frac{\partial u}{\partial y} = \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + Re^2\,(Gr\theta + Gm\emptyset) - \left(M^2 + \frac{1}{K}\right)u \tag{11}$$

$$\omega\frac{\partial \theta}{\partial t} - Re\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left[\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right] + Ec\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + Q\theta \tag{12}$$

$$\omega\frac{\partial \emptyset}{\partial t} - Re\frac{\partial \emptyset}{\partial y} = \frac{1}{Sc}\left(\frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2}\right) - Ch\,\emptyset \tag{13}$$

The boundary conditions (6) reduces to

$$u = 0,\quad \theta = 1 + \epsilon\cos(\pi z - t), \emptyset = 1,\quad at\ y = 0$$

$$u = 0,\quad \theta = 0,\quad \emptyset = 0,\qquad as\ y \to \infty \tag{14}$$

In order to establish a mathematical convenience of converging the solution at a finite point $\eta \to 1$, equations (11)-(14) should be transformed to a new system of coordinates $i.e$ $\boldsymbol{y}$ to $\boldsymbol{\eta}$. So, employing the transformation $\eta = 1 - e^{-y}$ on the equations (11)-(14), the following are obtained

$$\omega\frac{\partial u}{\partial t} - Re(1-n)\frac{\partial u}{\partial n} = \left[(1-n)^2\frac{\partial^2 u}{\partial n^2} - (1-n)\frac{\partial u}{\partial n} + \frac{\partial^2 u}{\partial z^2}\right] + Re^2(Gr\theta + Gm\emptyset) - \left(M^2 + \frac{1}{K}\right)u \tag{15}$$

$$\omega\frac{\partial \theta}{\partial t} - Re(1-n)\frac{\partial \theta}{\partial n} = \frac{1}{Pr}\left\{\left(1 + \frac{4}{3R}\right)\left((1-n)^2\frac{\partial^2 \theta}{\partial n^2} - (1-n)\frac{\partial \theta}{\partial n}\right) + \frac{\partial^2 \theta}{\partial z^2}\right\} + \left[(1-n)^2\left(\frac{\partial u}{\partial n}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + Q\theta \tag{16}$$

$$\omega\frac{\partial \emptyset}{\partial t} - Re(1-n)\frac{\partial \emptyset}{\partial n} = \frac{1}{Sc}\left[(1-n)^2\frac{\partial^2 \emptyset}{\partial n^2} - (1-n)\frac{\partial \emptyset}{\partial n} + \frac{\partial^2 \emptyset}{\partial z^2}\right] + Ch\,\emptyset \tag{17}$$

with the corresponding boundary conditions:

$$u = 0,\quad \theta = 1 + \epsilon\cos(\pi z - t),\ \emptyset = 1,\quad at\ n = 0 \tag{18}$$

$$u = 0,\quad \theta = 0,\quad \emptyset = 0,\qquad as\ n \to 1$$

In order to simplify the governing equations of the problem,

Let $$f(n,z,t) = f_0(n) + \varepsilon f_1(n)e^{i(\pi z - t)}\ , \tag{19}$$

Where f stands for $u\ \theta\ and\ \emptyset$

Substituting equation (19) into equations (15) to (17) and equating the like power of ε , we get the following zeroth and first order equations for the case of $\varepsilon = 0$, and $\varepsilon \neq 0$, respectively.

2.1 Zeroth Order Equations

$$(1-n)^2 \frac{d^2u_0}{dn^2} + (Re - 1)(1-n)\frac{du_0}{dn} - \left(M^2 + \frac{1}{K}\right)u_0 = -Re^2(Gr\theta_0 + Gm\emptyset_0) \quad (20)$$

$$\left(1+\frac{4}{3R}\right)(1-n)^2\frac{d^2\theta_0}{dn^2} + \left(RePr - 1 - \frac{4}{3R}\right)(1-n)\frac{d\theta_0}{dn} + QPr\theta_0 = -EcPr(1-n)^2\left(\frac{du_0}{dn}\right)^2 \quad (21)$$

$$(1-n)^2 \frac{d^2\emptyset_0}{dn^2} + (ReSc - 1)(1-n)\frac{d\emptyset_0}{dn} - Ch\,Sc\,\emptyset_0 = 0 \quad (22)$$

with the corresponding boundary conditions

$$u_0 = 0,\ \ \theta_0 = 1,\ \ \emptyset_0 = 1,\ \ \ at\ n = 0$$

$$u_0 = 0,\ \ \theta_0 = 0,\ \ \emptyset_0 = 0\ \ \ as\ n \to 1 \quad (23)$$

2.2 First Order Equations

$$(1-\eta)^2\frac{d^2u_1}{du^2} + (Re-1)(1-n)\frac{du_1}{dn} - \left(M^2 + \frac{1}{K} - i\omega + \pi^2\right)u_1 = Re^2(Gr\theta_1 + Gm\emptyset_1) \quad (24)$$

$$\left(1+\frac{4}{3R}\right)(1-n)^2\frac{d^2\theta_1}{dn^2} + \left(RePr - 1 - \frac{4}{3R}\right)(1-n)\frac{d\theta_1}{dn} - (\pi^2 - i\omega Pr - QPr)\theta_1 = 2EcPr\big((1-n)\big)^2\frac{\partial u_0}{\partial n}.\frac{\partial u_1}{\partial n} \quad (25)$$

$$(1-n)^2\frac{d^2\emptyset_1}{dn^2} + (ReSc-1)(1-n)\frac{d\emptyset_1}{dn} + \left(\pi^2 + i\omega Sc - Ch\,Sc\right)\emptyset_1 = 0 \quad (26)$$

with the corresponding boundary conditions

$$u_1 = 0,\ \ \theta_1 = 1, \emptyset_1 = 0,\ \ at\ n = 0$$

$$u_1 = 0,\ \ \theta_1 = 0,\ \ \emptyset_1 = 0,\ \ \ as\ n \to 1 \quad (27)$$

3. METHODS OF SOLUTION

The equations (20) and (21), (24) and (25) are coupled, non-linear differential equations whose exact solution is difficult to obtain, hence the problem is solved numerically, using the finite difference formulae of first and second order in to equations (20), (21), (22), (24), (25) and (26), the following system of equations are obtained in finite difference form:

3.1 Zeroth Order Equations (Finite Difference Form)

$$L_3u_0\,[i+1] - L_4u_0[i] + L_5u_0[i-1] = G[i] \quad (28)$$

$$M_1\theta_0[i+1] - M_2\theta_0[i] + M_3\theta_0[i-1] = H[i] \quad (29)$$

$$N_3\emptyset_0[i+1] - N_2\emptyset_0[i] + N_1\emptyset_0[i-1] = 0 \quad (30)$$

with the corresponding boundary conditions

$$u_0[i] = 0,\ \ \theta_0[i] = 1,\ \ \emptyset_0[i] = 1, \quad for\ \ i = 0$$

$$u_0[i] = 0,\ \ \theta_0[i] = 0,\ \ \emptyset_0[i] = 0, \quad for\ i = 10 \tag{31}$$

Here, i stands for the plate divisions with size h=0.1 and $\eta = i\ h$.

Where,

$$L_1 = \ (Re - 1), \quad L_2 = M^2 + \frac{1}{K},$$

$$L_3 = 2(1 - ih)^2 + L_1(1 - ih)h, \ \ L_4 = 4(1 - ih)^2 + 2L_2h^2$$

$$L_5 = 2(1 - ih)^2 - L_1(1 - ih)h, \qquad G[i] = \ -2Re^2\left[Gr\theta_0[i] + Gm\ \theta_0[i]\right]h^2$$

$$M_1 = 2\left(1 + \frac{4}{3R}\right)(1 - ih)^2 - \left(Re\ Pr - 1 - \frac{4}{3R}\right)(1 - ih)h$$

$$M_2 = 4\left(1 + \frac{4}{3R}\right)(1 - ih)^2 - 2Q\ Pr\ h^2$$

$$M_3 = 2\left(1 + \frac{4}{3R}\right)(1 - ih)^2 + \left(Re\ Pr - 1 - \frac{4}{3R}\right)(1 - ih)h$$

$$H[i] = -Ec\ Pr(1 - ih)^2\left[\frac{u_0[i + 1] - u_0[i - 1]}{2h}\right]^2$$

$$N_1 = 2(1 - ih)^2 - (Re\ Sc - 1)(1 - ih)h, \ \ N_2 = (4(1 - ih)^2 + 2Ch\ Sc\ h^2)$$

$$N_3 = 2(1 - ih)^2 + (Re\ Sc - 1)(1 - ih)h$$

3.2 First Order Equations (Finite Difference Form)

$$J_1u_1[i + 1] - J_2u_1[i] + J_2u_1[i - 1] \ = J[i] \tag{32}$$

$$P_1\theta_1[i + 1] - P_2\theta_1[i] + P_3\theta_1[i - 1] \ = P[i] \tag{33}$$

$$F_1\emptyset_1[i + 1] - F_2\emptyset_1[i] + F_3\emptyset_1[i - 1] \ = 0 \tag{34}$$

with corresponding boundary conditions :

$$u_1[i] = 0,\ \ \theta_1[i] = 1,\ \ \emptyset_1[i] = 1, \quad for\ \ i = 0$$

$$u_1[i] = 0, \ \ \theta_1[i] = 0,\ \ \emptyset_1[i] = 0, \quad for\ i = 10 \tag{35}$$

Here, i stands for the plate divisions with size h=0.1 and $\eta = i\ h$.

Where,

$$J_1 = 2(1 - ih)^2 + h(Re - 1)(1 - n), \ \ J_2 = 4(1 - ih)^2 + 2\left(M^2 + \frac{1}{K} - i\omega - \pi^2\right)h^2$$

$$J_3 = 2h^2\left(M^2 + \frac{1}{K} - i\omega + \pi^2\right), \qquad J[i] = -2Re^2h^2\left[Gr\theta_1[i] + Gm\emptyset_1[i]\right]$$

$$P_1 = 2\left(1 + \frac{4}{3R}\right)(1 - ih)^2 - \left(Re\ Pr - 1 - \frac{4}{3R}\right)(1 - ih)h$$

$$P_2 = \left[4\left(1+\frac{4}{3R}\right)(1-ih)^2 + 2(\pi^2 - i\omega Pr - QPr)h^2\right]$$

$$P_3 = 2\left(1+\frac{4}{3R}\right)(1-ih)^2 + \left(Re\ Pr - 1 - \frac{4}{3R}\right)(1-ih)h$$

$$P[i] = -Ec\ Pr(1-ih)^2(u_0[i+1] - u_0[i-1])(u_1[i+1] - u_1[i-1])$$

$$F_1 = 2(1-ih)^2 - (Re\ Sc - 1)(1-ih)h$$

$$F_2 = 4(1-ih)^2 + 2(\pi^2 + i\omega Sc - Ch\ Sc)h^2,\ \ F_3 = 2(1-ih)^2 - (Re\ Sc - 1)(1-ih)h\ .$$

Equations (28), (29), (30) and (32), (33), (34) with corresponding boundary conditions have been solved by using Gauss-seidel iteration method and simulation is carried out by coding in C-Program. To prove the convergence of finite difference scheme, the computation is carried out by slightly changed values of h and the iterations on until a tolerance 10^{-8} is attained. No significant change was observed in the values of $u, \theta\ and\ \phi$. Thus, it is concluded that the finite difference scheme is convergent and stable.

4. RESULTS AND DISCUSSION

In order to get physical insight into the problem, the numerical calculations for the distribution of the velocity, temperature, concentration, across the boundary layer for various values of the parameter have been carried out. The effects of the main controlling parameters as they appear in the governing equations are discussed in the current section. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 and Pr = 7.0, which represent air and water at temperature 20ºC and one atmosphere pressure, respectively. For the physical significance**,** in the present problem, the attention is being paid to the real parts of complex quantities only.

Fig. 1 displays the effect of magnetic parameter M on velocity field u in the presence of radiation. It can be inferred from figure that an increase in M leads to decrease in the velocity. The presence of magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied normal to the fluid flow. This type of resistive force tends to slow down the flow field. Further, it is interesting to note that the velocity of the fluid flow decreases in the presence of radiation this due to the fact that an increase in the radiation parameter R the rate of radiative heat, transferred to the fluid decreases and consequently the fluid temperature and hence the velocity of its particles also decreases.

Figs. 2 and 10 depict the velocity and temperature profiles, respectively for various values of heat generation/absorption parameter. It is evident from the figures that the temperature and velocity increase with an increase in the heat generation parameter (Q). This result qualitatively agrees with expectation since the effect of heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid and also increasing its velocity. It is also observed that temperature and velocity of the fluid decrease in the presence of heat absorption as heat absorption is to decrease the rate of heat transfer to the fluid.

Figs. 3 and 4 show the velocity field u for various values of Grashof number (Gr) and modified Grashof number (Gm), respectively. It is observed from the figures that an increase in Gr and Gm leads to increase in the velocity of the flow. This is due to the fact that with the increasing values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. It is also observed that as the values of Gr (or) Gm increases, the peak value of the velocity increases rapidly near the wall of the plate and then decay to the free stream velocity. From Fig. 5, an important observation noted that an increase in permeability parameter leads to increase in the velocity of the flow as the degrees of moment of particles in the fluid becomes maximum. It is also observed that in the presence of radiation, velocity of the fluid decreases as the rate of radiative heat, transferred to the fluid decreases.

Figs. 6 and 9 are drawn for different values of radiation parameter R on velocity and temperature field respectively. It is revealed that the temperature and velocity decrease as the radiation parameter increases. This result can be explained by the fact that a decrease in the radiation parameter R= $\frac{k a_R}{4\sigma^* T_\infty^3}$, forgiven k and T_∞, means a decrease in the Roseland radiation absorbptivity (a_R). In view of equations (3) and (5), it is concluded that the divergence of the radiation heat flux $\partial q_r / \partial y^*$, increases as a_R decreases and this means that the rate of radiative heat, transferred to the fluid increases and consequently the fluid temperature and hence the velocity of its particles also increases.

Figs. 7 and 8 show the temperature profile for various values of Pr and Ec respectively. A comparative study of the graph reveals that the presence of heavier Prandtl number in the flow field is found to decelerate temperature at all points. This is due to the physical fact that a fluid with high Prandtl number has a relatively low thermal conductivity which results in the reduction of the thermal boundary layer. The analysis of Fig. 8 reveals that an increase in Eckert number (Ec) leads to increase in the temperature distribution, due to the frictional heating.

The effects of Schmidt number (Sc) and Chemical reaction parameter (Ch) on the concentration field is exhibited by the curves shown in Figs. 11 and 12 respectively. From these it is observed that for increasing values of the Sc and Ch there is a fall in the concentration of the fluid. Physically, the increase of Sc means decrease of molecular diffusivity (D) those results in decrease of concentration boundary layer. Therefore, the concentration of the species is higher for small values of Sc and lower for larger values of Sc.

The results obtained are compared with those of K.D.Singh, Rakesh kumar [44] in the absence of radiation parameter R for velocity and temperature profiles. The deviation found between previous and present results is 10^{-8}. Therefore the proposed technique, present in the paper is an efficient algorithm with assured convergence. It gives an indication of high degree of coincidence with realistic physical phenomenon and earlier reported studies.

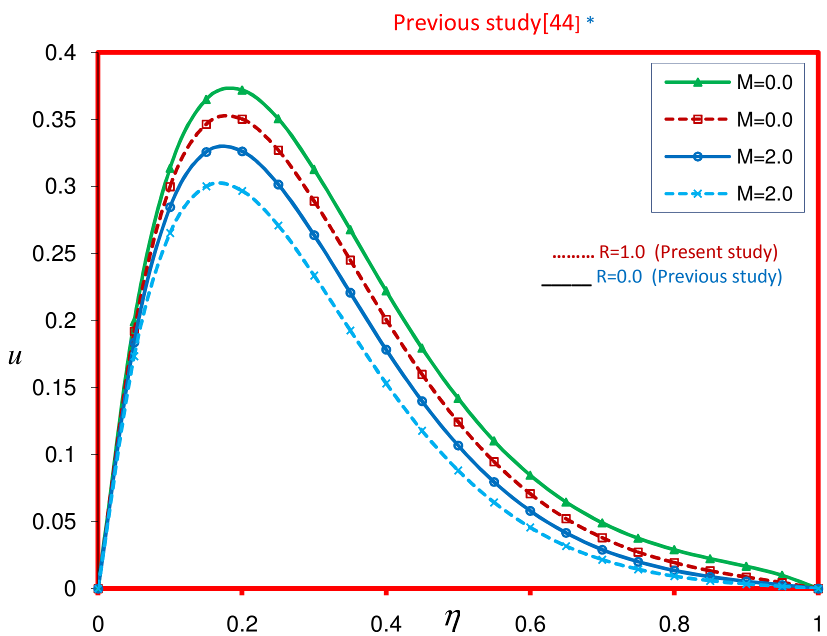

Fig. 1. Effect of Magnetic parameter (M) on velocity field u in the presence of radiation
(Gr=5.0, Gm=5.0, K=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

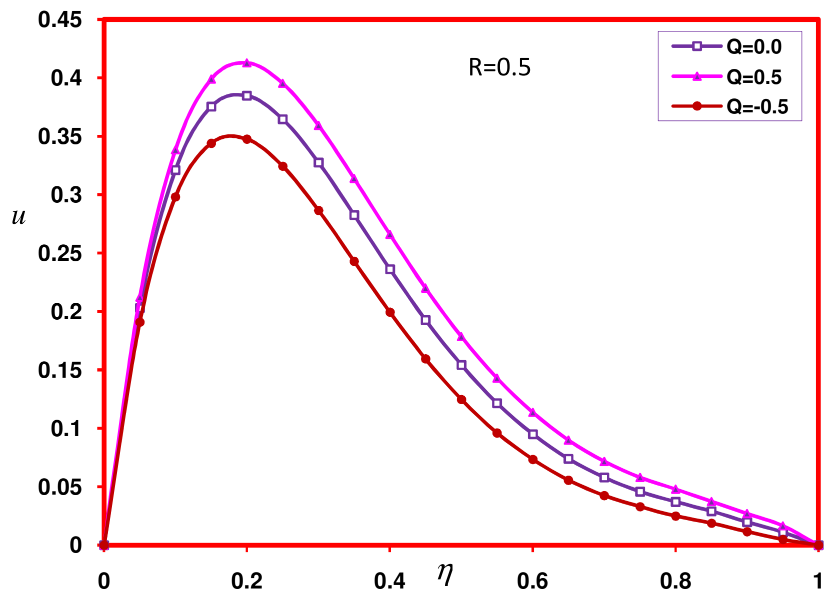

Fig. 2. Effect of heat generation/absorption (Q) on velocity field u
(Gr=5.0, Gm=5.0, R=0.5, K=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

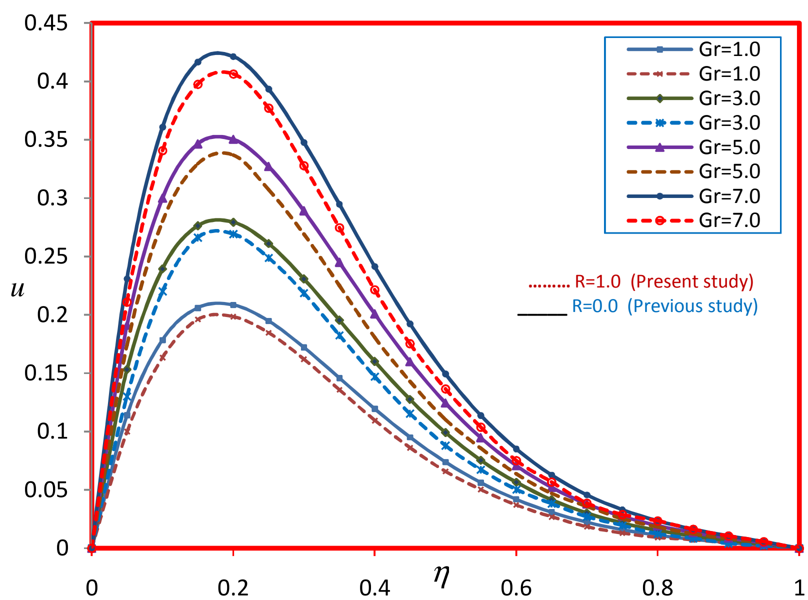

Fig. 3. Effect of Grash of number (Gr) on velocity field u in the presence of radiation
(Gm=5.0, K=0.5, Q=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

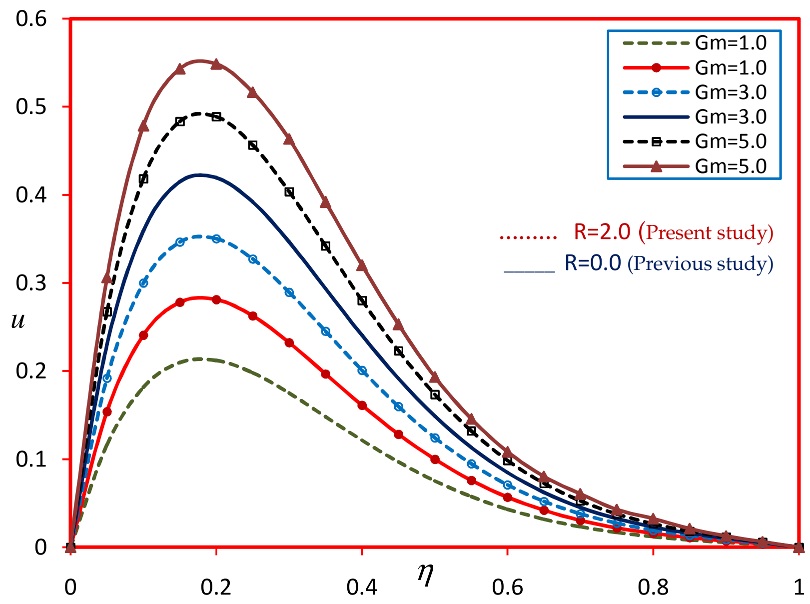

Fig. 4. Effect of modified Grashof number (Gm) on velocity field u in the presence of radiation
(Gr=5.0, K=0.5, Q=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

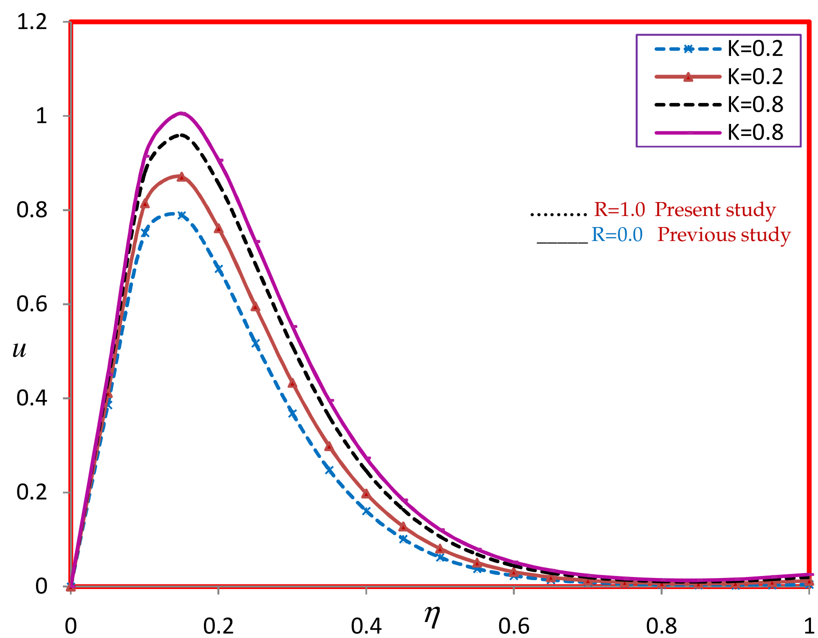

Fig. 5. Effect of permeability (K) on velocity field u in the presence of radiation
(Gr=5.0, Gm=5.0, Q=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

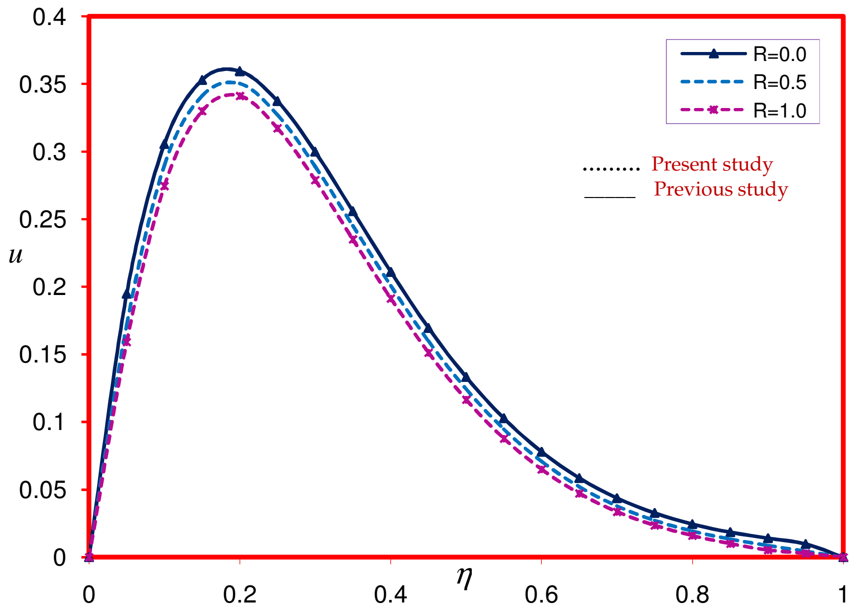

Fig. 6. Effect of radiation (R) on transient velocity in the presence of radiation
(Gr=5.0, Gm=5.0, K=0.5, Q=0.5, Ec=0.2, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, ωt=π/2 and ε=0.01)

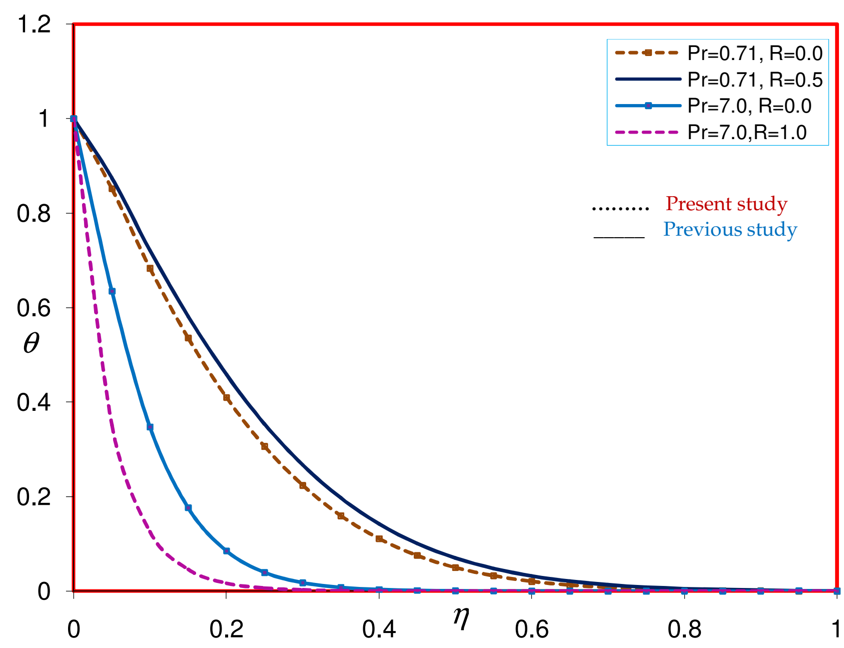

Fig. 7. Effect of Prandtl number (Pr) on transient temperature in the presence of radiation
(*Gr=5.0, Gm=5.0, K=0.5, Q=0.5, Ec=0.2, M=1.0, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)*

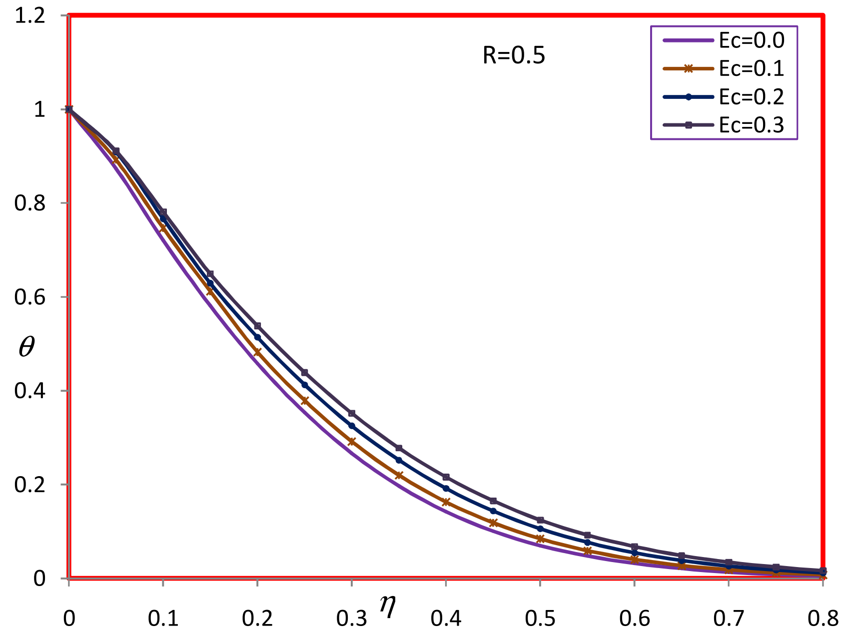

Fig. 8. Effect of Eckert number (Ec) on transient temperature
(Gr=5.0, Gm=5.0, K=0.5, Q=0.5, R=0.5, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

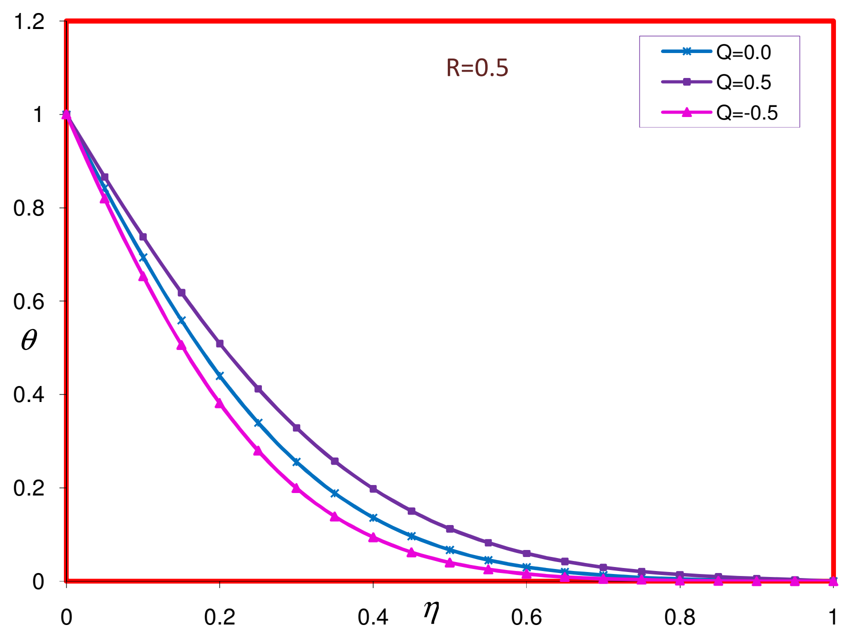

Fig. 9. Effect of radiation (R) on transient temperature
(Gr=5.0, Gm=5.0, K=0.5, Q=0.5, Ec=0.5, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

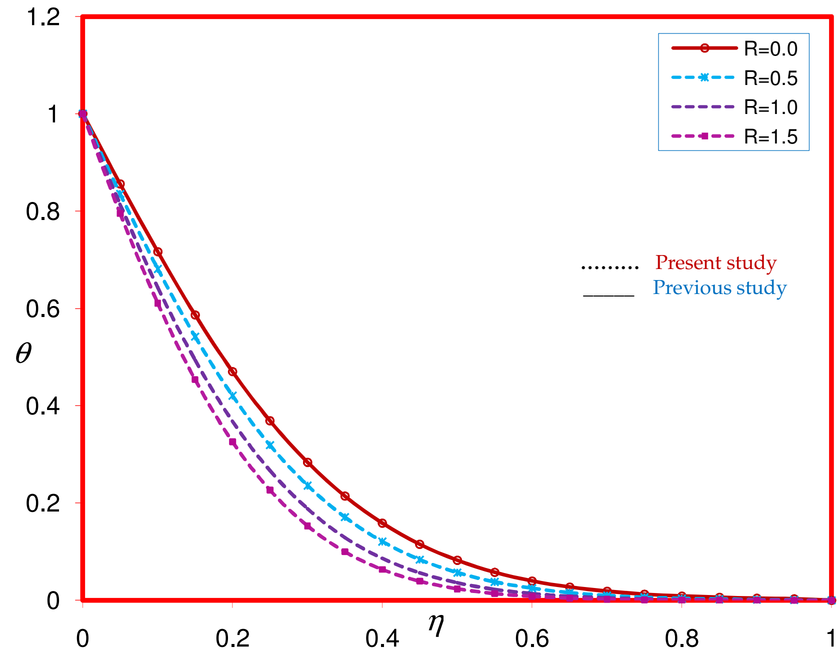

Fig. 10. Effect of heat generation/absorption (Q) on transient temperature
(Gr=5.0, Gm=5.0, K=0.5, Ec=0.5, M=1.0, Pr=0.71, Sc=0.22, Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

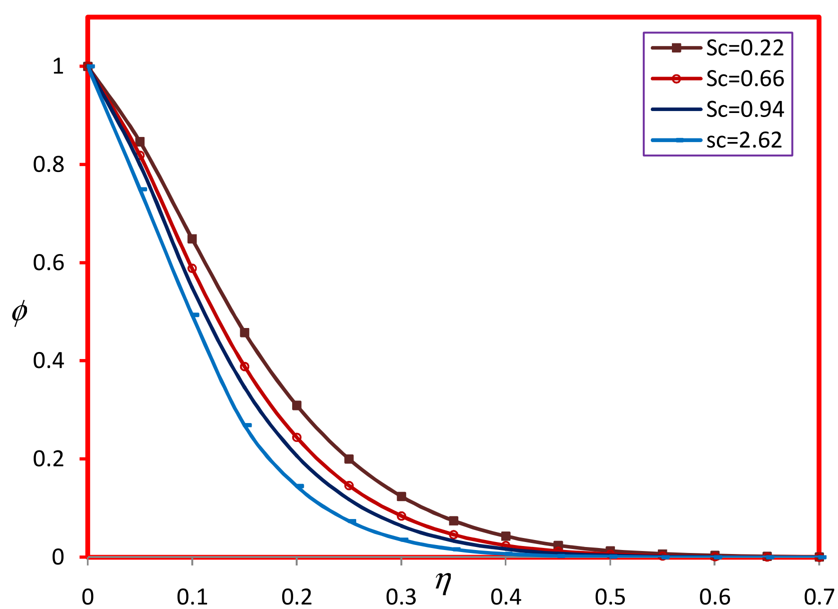

Fig. 11. Effect of Schmidth number (Sc) on transient concentration
(Ch=0.5, Re=2.0, t=π/2 and ε=0.01)

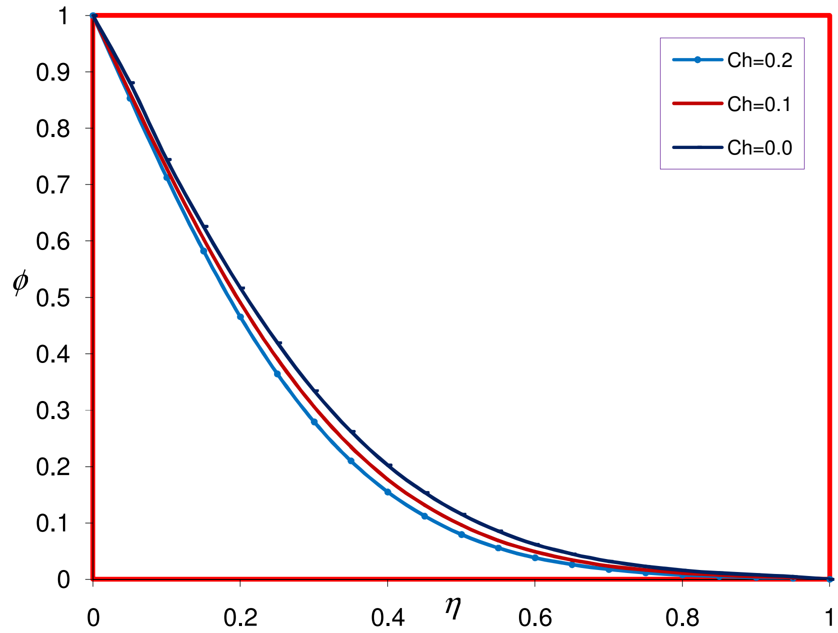

Fig. 12. Effect of chemical reaction (Ch) on transient concentration
(Sc=0.22, Re=2.0, t=π/2 and ε=0.01)

5. CONCLUSION

- Temperature and velocity decrease for increasing values of radiation parameter, i.e., for decreasing values radiation parameter, the fluid temperature and velocity of its particles also increases as the rate of radiative heat, transferred to the fluid increases.
- The temperature and velocity of the fluid increase for increasing values of heat generation parameter (Q), as the effect of heat generation is to increase the rate of heat transport to the fluid. Temperature and velocity of the flow decrease in the presence of heat absorption parameter.
- Temperature of the fluid increases for increasing values of Eckert number (Ec), due to the frictional heating between the plate and the fluid.
- In order to assess the validity of present method, the results obtained are compared with those of K.D.Singh, Rakesh kumar [44] in the absence of radiation parameter R for velocity and temperature profiles. The comparisons in all the cases are found to be good agreement with physical phenomenon and previous results [44] and hence it is concluded that proposed technique is an efficient numerical algorithm with assured convergence.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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