Journal of Advances in Mathematics and Computer Science

30(4): 1-15, 2019; Article no.JAMCS.44846

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Fully Implicit Five-quarters Computational Algorithms of Order Five for Numerical Approximation of Second Order IVPs in ODEs

L. A. Ukpebor[1], E. O. Omole[2*] and L. O. Adoghe[1]

[1]Department of Mathematics, Ambrose Alli University, Ekpoma, Edo State, Nigeria.
[2]Department of Mathematical Sciences, Federal University of Technology, Akure, Ondo State, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Author LAU designed the study and wrote the first draft of the manuscript. Author LOA managed the analyses and the literature searches while author EOO managed the numerical implementation of the study. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2019/44846
Editor(s):
(1) Dr. Zhenkun Huang, Professor, School of Science, Jimei University, China.
Reviewers:
(1) W. Obeng-Denteh, Kwame Nkrumah University of Science and Technology, Ghana.
(2) Adeyeye Oluwaseun, Universiti Utara Malaysia, Malaysia.
(3) Aliyu Bhar Kisabo, Nigeria.
Complete Peer review History: http://www.sdiarticle3.com/review-history/44846

Received: 07 September 2018
Accepted: 30 November 2018
Published: 11 February 2019

Original Research Article

Abstract

This research paper proposes a fully implicit five-quarters computational algorithms of order five for Numerical Approximation of Second order IVPs in ODEs. The method is consistent, convergent, zero stable and A-stable. The method solved second order ordinary differential equations efficiently and accurately. It has a low error constant and gives better approximation than some existing methods.

Keywords: Fully-implicit; five-quarters; second order; approximation; A-stable; order; dynamic model; IVPs; ODEs.

Subject Classification: 65L05; 65L06; 65L20

**Corresponding author: E-mail: omolez247@gmail.com;*

1 Introduction

This paper considered the numerical solution of second order initial value problem of the form

$$y''=f\left(x,y,y'\right)\ ,\ \ y\left(x_0\right)=y_0,\ y'\left(x_0\right)=y_1 \tag{1}$$

Where f is continuous within the interval of integration. Equation (1) is of interest to researchers because of its wide application in several field of studies, such as engineering, biology, control theory and other real life problem, hence the study of the methods of its solution is very important. Hence, authors proposed methods with different basis functions and among them are [1,2,3,4,5,6,7,8,9,10,11,12,13] to mention a few.

Direct block method of solving higher order ordinary differential equation has been discussed by many authors and they came into conclusion that directs methods are more convenient, efficient and accurate than the method of reduction to systems of first order ordinary differential equation [2]. Among the authors that proposed the direct methods are [2,4,5,6]. This block method has the properties of Runge-kutta method for being self-starting and does not require development of separate predictors or starting values. Among these authors are [1,2,3,7,9,11]. Block method was found to be cost effective and give better approximation.

The aim of this paper is to propose a class of Fully Implicit five-quarters Computational Algorithms of order five for Numerical Approximation of Second order IVPs in ODEs with constant step-size which is implemented in block mode.

The paper is organized as followed: Section 2 considers the mathematical formulation of the method. Section 3 considers the analysis of the basic properties of the method. Section 4 considers the Region of absolute stability of our method. Section 5 considers the application of the derived method to solve some second order Ordinary Differential Equations and conclusion.

2 Mathematical Formulation of Five-quarters Step Method

Taylor series expansion of exponential function is adopted as a basis function for the approximation of (1)

$$Y(x)=\sum_{j=0}^{r+s-1} w_j P_j(x) \tag{2}$$

where $P_j(x)=\dfrac{x^j}{j!}$ and $w_j\,'s$ are the coefficients to be determined and a polynomial of degree $r+s-1$. The 4-Point hybrid Computational method is constructed by imposing the following conditions on (2).

$$Y(x_{n+j})=y_{n+j,}j=0,1,2,...,r-1 \tag{3}$$

$$Y''(x_{n+j})=f_{n+j,}j=0,1,2,...,s-1 \tag{4}$$

Putting (1) into (4) gives

$$f(x,y,y')=\left[P_j(x)\right]'' \tag{5}$$

Here, a step-length of $k=\dfrac{5}{4}$ with constant step size $h=\dfrac{5}{16}$

Interpolating (3) at $x = x_{n+\frac{5}{8}}, x_{n+\frac{15}{16}}$ and collocating (4) at $x = x_n, x_{n+\frac{5}{16}}, x_{n+\frac{5}{8}}, x_{n+\frac{15}{16}}, x_{n+\frac{5}{4}}$ gives a system of non-linear equation of the form

$$AX = B \tag{6}$$

$$A = [w_0, w_1, w_2, w_3, w_4, w_5, w_6]^T$$

where $B = \left[y_n, y_{n+\frac{5}{16}}, f_n, f_{n+\frac{5}{16}}, f_{n+\frac{5}{8}}, f_{n+\frac{15}{16}}, f_{n+\frac{5}{4}} \right]^T$

$$X = \begin{bmatrix} 1 & x^1_{n+\frac{5}{8}} & \frac{x^2_{n+\frac{5}{8}}}{2!} & \frac{x^3_{n+\frac{5}{8}}}{3!} & \frac{x^4_{n+\frac{5}{8}}}{4!} & \frac{x^5_{n+\frac{5}{8}}}{5!} & \frac{x^6_{n+\frac{5}{8}}}{6!} \\ 1 & x^1_{n+\frac{15}{16}} & \frac{x^2_{n+\frac{15}{16}}}{2!} & \frac{x^3_{n+\frac{15}{16}}}{3!} & \frac{x^4_{n+\frac{15}{16}}}{4!} & \frac{x^5_{n+\frac{15}{16}}}{5!} & \frac{x^6_{n+\frac{15}{16}}}{6!} \\ 0 & 0 & 1 & x_n & \frac{x_n^2}{2!} & \frac{x_n^3}{3!} & \frac{x_n^4}{4!} \\ 0 & 0 & 1 & \frac{x_{n+\frac{15}{16}}}{1!} & \frac{x^2_{n+\frac{15}{16}}}{2!} & \frac{x^3_{n+\frac{15}{16}}}{3!} & \frac{x^4_{n+\frac{15}{16}}}{4!} \\ 0 & 0 & 1 & \frac{x_{n+\frac{5}{8}}}{1!} & \frac{x^2_{n+\frac{5}{8}}}{2!} & \frac{x^3_{n+\frac{5}{8}}}{3!} & \frac{x^4_{n+\frac{5}{8}}}{4!} \\ 0 & 0 & 1 & \frac{x_{n+\frac{15}{16}}}{1!} & \frac{x^2_{n+\frac{15}{16}}}{2!} & \frac{x^3_{n+\frac{15}{16}}}{3!} & \frac{x^4_{n+\frac{15}{16}}}{4!} \\ 0 & 0 & 1 & \frac{x_{n+\frac{5}{4}}}{1!} & \frac{x^2_{n+\frac{5}{4}}}{2!} & \frac{x^3_{n+\frac{5}{4}}}{3!} & \frac{x^4_{n+\frac{5}{4}}}{4!} \end{bmatrix}$$

Solving (6) for the $w_i's$ using Gaussian elimination method or Crammer's rule and substitute the values into (2), gives a continuous hybrid computation method of the form;

$$y(x) = \alpha_{\frac{5}{8}} y_{n+\frac{5}{8}} + \alpha_{\frac{15}{16}} y_{n+\frac{15}{16}} + \mathrm{h}^2 \left(\sum_{i=0}^{k} \sigma_i(x) f_{n+i} + \sigma_k(x) f_{n+k} \right), i = 0, \frac{5}{16}, \frac{5}{8}, \frac{15}{16}, \; k = \frac{5}{4} \tag{7}$$

The coefficient of $y_{n+i}, i = \frac{5}{16}, \frac{5}{8}, \frac{15}{16}, \frac{5}{4}$ and $f_{n+i}, i = 0(\frac{5}{16}), \frac{5}{4}$

$$\left\{\begin{array}{l}\alpha_{\frac{5}{8}}=\dfrac{-1}{5}(16t-15)\\ \alpha_{\frac{15}{16}}=\dfrac{2}{5}(8t-5)\\ \sigma_0=\left(\dfrac{h^2}{115200000}\right)\left((16t-15)(8t-5)(131072t^4-409600t^3+403200t^2-130000t+10625)\right)\\ \sigma_{\frac{5}{16}}=\left(\dfrac{-h^2}{28800000}\right)\left((16t-15)(8t-5)(131072t^4-348160t^3+211200t^2+54000t-39375)\right)\\ \sigma_{\frac{5}{8}}=\left(\dfrac{h^2}{19200000}\right)\left((16t-15)(8t-5)(131072t^4-286720t^3+83200t^2+58000t+41875)\right)\\ \sigma_{\frac{15}{16}}=\left(\dfrac{-h^2}{28800000}\right)\left((16t-15)(8t-5)(131072t^4-225280t^3+19200t^2+2000t-8125)\right)\\ \sigma_{\frac{5}{4}}=\left(\dfrac{h^2}{115200000}\right)\left((16t-15)(8t-5)(131072t^4-163840t^3+19200t^2+6000t-1875)\right)\end{array}\right\}$$

Where $t=\dfrac{x-x_n}{h},\ \dfrac{dt}{dx}=\dfrac{1}{h},\ y_{n+i}=y(x_n+ih),\ f_{n+i}=f(x_n+ih),\ y'(x_n+ih)$ (8)

Solving (7) for the independent solution at the off grip points give the continuous block method.

$$y(x)=\sum_{i=0}^{k}\frac{(ih)^n}{n!}(y_n)^n+\mathrm{h}^2\left(\sum_{i=0}^{k}\tau_i(x)f_{n+i}+\tau_k(x)f_{n+k}\right),\ i=0,\frac{5}{16},\frac{5}{8},\frac{15}{16},\ k=\frac{5}{4} \tag{9}$$

The coefficient of f_{n+i} and f_{n+k} give;

$$\left\{\begin{array}{l}\tau_0=\dfrac{1}{14400000}\left((32t-25)(393216t^4-1228800t^3+1280000t^2-500000t+59375)\right)\\ \tau_{\frac{5}{16}}=\dfrac{-1}{3600000}\left((12582912t^5-44236800t^4+53248000t^3-23040000t^2+1490625)\right)\\ \tau_{\frac{5}{8}}=\dfrac{1}{2400000}\left((12582912t^5-39321600t^4+38912000t^3-11520000t^2-503125)\right)\\ \tau_{\frac{15}{16}}=\dfrac{-1}{3600000}\left((12582912t^5-34406400t^4+2867200t^3-7680000t^2+221875)\right)\\ \tau_{\frac{5}{4}}=\dfrac{1}{14400000}\left((12582912t^5-29491200t^4+22528000t^3-5760000t^2+103125)\right)\end{array}\right\} \tag{10}$$

Evaluating (9) at $t=0(\frac{5}{16}),\frac{5}{4}$ give a discrete block formula of the form

$$A^{(o)}Y_m^{(i)} = \sum_{i=0}^{k} h^i e_i y_n^{(i)} + h^2 d_i f(y_n) + h^2 b_i f(Y_m), i = 0,1 \tag{11}$$

where

$$Y_m = \begin{bmatrix} y_{n+\frac{5}{16}} \\ y_{n+\frac{5}{8}} \\ y_{n+\frac{15}{16}} \\ y_{n+\frac{5}{4}} \end{bmatrix},\ f(Y_m) = \begin{bmatrix} f_{n+\frac{5}{16}} \\ f_{n+\frac{5}{8}} \\ f_{n+\frac{15}{16}} \\ f_{n+\frac{15}{4}} \end{bmatrix},\ y_n^{(i)} = \begin{bmatrix} y^{(i)}_{n+\frac{5}{4}} \\ y^{(i)}_{n+\frac{15}{16}} \\ y^{(i)}_{n+\frac{5}{8}} \\ y_n^{(i)} \end{bmatrix}, f(y_n) = \begin{bmatrix} f_{n-\frac{5}{4}} \\ f_{n-\frac{15}{16}} \\ f_{n-\frac{5}{8}} \\ f_n \end{bmatrix},\ A^0 = 5\,X\,5$$

Identity Matrix

When i=0

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},\quad e_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{5}{16} \\ 0 & 0 & 0 & \frac{5}{8} \\ 0 & 0 & 0 & \frac{15}{16} \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix}, d_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{1835}{73728} \\ 0 & 0 & 0 & \frac{265}{4608} \\ 0 & 0 & 0 & \frac{735}{8192} \\ 0 & 0 & 0 & \frac{35}{288} \end{bmatrix},\quad b_0 = \begin{bmatrix} \frac{75}{2048} & \frac{-235}{12288} & \frac{145}{18432} & \frac{-35}{24576} \\ \frac{5}{32} & \frac{-25}{768} & \frac{5}{288} & \frac{-5}{1536} \\ \frac{585}{2048} & \frac{135}{4096} & \frac{75}{2048} & \frac{-45}{8192} \\ \frac{5}{12} & \frac{5}{48} & \frac{5}{36} & 0 \end{bmatrix},$$

When i=1

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},\quad d_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{2304} \\ 0 & 0 & 0 & \frac{29}{288} \\ 0 & 0 & 0 & \frac{27}{256} \\ 0 & 0 & 0 & \frac{7}{72} \end{bmatrix},\quad b_1 = \begin{bmatrix} \frac{323}{1152} & \frac{-11}{96} & \frac{53}{1152} & \frac{-19}{2304} \\ \frac{31}{72} & \frac{1}{12} & \frac{1}{72} & \frac{-1}{288} \\ \frac{51}{128} & \frac{9}{32} & \frac{21}{128} & \frac{-3}{256} \\ \frac{4}{9} & \frac{1}{6} & \frac{4}{9} & \frac{7}{72} \end{bmatrix}$$

3 Basic Properties of the Developed Method

3.1 Order and error constant of the block method

Let the linear Operator defined on the method be $\varsigma[y(x);h],$ where

$$\Delta[y(x);h] = A^{(o)}Y_m^{(i)} - \sum_{i=0}^{k} \frac{jh}{i} y_n^{'} - h^{(2-i)}[d_i f(y_n) + b_i F(y_m)], \tag{12}$$

Expanding the form Y_m and $F(y_m)$ in Taylor Series and comparing coefficients of h, we obtained

$$\Delta[y(x);h]=C_0y(x)+C_1hy'(x)+\ldots\ C_ph^py^p(x)+C_{p+1}h^{p+1}y^{p+1}(x)+C_{p+2}h^{p+2}y^{p+2}(x)+\ldots \tag{13}$$

Theorem 1: The linear operator and the associated block method are said to be of order p if $C_0 = C_1 = \ldots C_p = C_{p+1} = 0, C_{p+2} \neq 0 \quad C_{p+2}$ is called the error constant. It implies that the local truncation error is given by

$$T_{n+k} = C_{p+2}h^{p+2}y^{p+2}(x) + O(h^{p+3}) \tag{14}$$

Expanding the block in Taylor Series expansion gives.

$$\begin{cases}
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{16}\right)^j h^j}{j!}y_n^{(j)} - y_n - \frac{5}{16}hy_n^{(1)} - \sum_{j=0}^{\infty}\frac{h^{(j+2)}}{j!}y_n^{(j+2)}\left[\frac{75}{2048}\left(\frac{5}{16}\right)^j - \frac{235}{12288}\left(\frac{5}{8}\right)^j + \frac{145}{18432}\left(\frac{15}{16}\right)^j - \frac{35}{24576}\left(\frac{5}{4}\right)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{8}\right)^j h^j}{j!}y_n^{(j)} - y_n - \frac{5}{8}hy_n^{(1)} - \sum_{j=0}^{\infty}\frac{h^{(j+2)}}{j!}y_n^{(j+2)}\left[\frac{5}{32}\left(\frac{5}{16}\right)^j - \frac{25}{768}\left(\frac{5}{8}\right)^j + \frac{5}{288}\left(\frac{15}{16}\right)^j - \frac{5}{1536}\left(\frac{5}{4}\right)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{15}{16}\right)^j h^j}{j!}y_n^{(j)} - y_n - \frac{15}{16}hy_n^{(1)} - \sum_{j=0}^{\infty}\frac{h^{(j+2)}}{j!}y_n^{(j+2)}\left[\frac{585}{2048}\left(\frac{5}{16}\right)^j + \frac{135}{4096}\left(\frac{5}{8}\right)^j + \frac{75}{2048}\left(\frac{15}{16}\right)^j - \frac{45}{8192}\left(\frac{5}{4}\right)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{4}\right)^j h^j}{j!}y_n^{(j)} - y_n - \frac{5}{4}hy_n^{(1)} - \sum_{j=0}^{\infty}\frac{h^{(j+2)}}{j!}y_n^{(j+2)}\left[\frac{5}{12}\left(\frac{5}{16}\right)^j + \frac{5}{48}\left(\frac{5}{8}\right)^j + \frac{5}{36}\left(\frac{15}{16}\right)^j - 0\left(\frac{5}{4}\right)^j\right]
\end{cases}$$

Comparing the coefficients of h, the order of the block is $p = 5$ with error constant

$$C_{p+2} = \left[\frac{1671875}{541165879296}, \frac{15625}{2113929216}, \frac{703125}{60129542144}, \frac{15625}{1056964608}\right]^T$$

3.2 Consistency

In numerical analysis, it is important that a linear multistep method satisfied the necessary and sufficient conditions. A numerical method is said to be consistent if the following conditions are satisfied.

i. The order of the methods must be greater than or equal to 1 i.e. p≥1.
ii. $\sum_{j=0}^{k}\alpha_j = 0, \alpha_j$'s are the coefficients of the first characteristic polynomial
iii. $\rho(r) = \rho'(r) = 0$ where $r = 1$, root of the characteristics polynomial
iv. $\rho^{ii}(r) = 2!\sigma(r)$ for $r = 1$, σ(r) is the second characteristic polynomial.

According to Lambert (1973) the first condition is a sufficient condition for the associated block method to be consistent. Hence the developed method is consistent.

3.3 Zero stability of the method

The general form of block method is given as

$$A^{(0)}Y_m = A^{(i)}Y_{m-i} + h^{mu}[B^iY_m + B^iY_{m-i}] \tag{15}$$

Using (11) gives

$$\left\|\left[\lambda A^{(0)} - A^{(i)}\right]\right\| = \left[z\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} - \begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}\right] = 0$$

$$\lambda^4 - \lambda^3 = 0, \lambda = 0, 0, 01$$

Since no root has modulus greater than one and $|\lambda| = 1$ is simple, hence the block method is zero stable in the $h \to 0$

4 Region of Absolute Stability of the Proposed Method

According to Olabode and Omole [3], Ibijola et al. [5], The stability matrix is expressed as

$$M(z) = V + zB(M - zA)^{-1}U \tag{16}$$

together with the stability function

$$p(\eta, z) = \det(\eta I - M(z)) \tag{17}$$

We express block method (11) in the form

$$\begin{bmatrix} Y \\ \hline Y_{i+1}\end{bmatrix} = \begin{bmatrix} A & U \\ \hline B & V\end{bmatrix}\begin{bmatrix} h^2 f(y) \\ \hline Y_{i-1}\end{bmatrix} \tag{18}$$

where

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{1835}{73728} & \frac{75}{2048} & \frac{-235}{12288} & \frac{145}{18432} & \frac{-35}{24576} \\ \frac{265}{4608} & \frac{5}{32} & \frac{-25}{768} & \frac{5}{288} & \frac{-5}{1536} \\ \frac{735}{8192} & \frac{585}{2048} & \frac{135}{4096} & \frac{75}{2048} & \frac{-45}{8192} \\ \frac{35}{288} & \frac{5}{12} & \frac{5}{48} & \frac{5}{36} & 0 \end{bmatrix}, \quad B=\begin{bmatrix} \frac{1835}{73728} & \frac{75}{2048} & \frac{-235}{12288} & \frac{145}{18432} & \frac{-35}{24576} \\ \frac{35}{288} & \frac{5}{12} & \frac{5}{48} & \frac{5}{36} & 0 \end{bmatrix}$$

$$V=\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$U=\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, M=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, Y=\begin{bmatrix} y_n \\ y_{n+\frac{5}{16}} \\ y_{n+\frac{5}{8}} \\ y_{n+\frac{15}{16}} \\ y_{n+\frac{5}{4}} \end{bmatrix}, \quad f(y)=\begin{bmatrix} f_n \\ f_{n+\frac{5}{16}} \\ f_{n+\frac{5}{8}} \\ f_{n+\frac{15}{16}} \\ f_{n+\frac{5}{4}} \end{bmatrix}, Y_{i-1}=\begin{bmatrix} y_{n+\frac{5}{16}} \\ y_n \end{bmatrix}, Y_{i+1}=\begin{bmatrix} y_{n+\frac{5}{16}} \\ y_{n+\frac{5}{4}} \end{bmatrix} \tag{19}$$

The elements of the matrices A, B, U and V are substituted and computing the stability function with Maple software yield the stability polynomial of the method which is then plotted in MATLAB environment to produce the required absolute stability region of the method. The graph is shown by the figure below, shows that the Method is A-Stable and the plot covers a large region of the complex plane $z\varepsilon C^n$

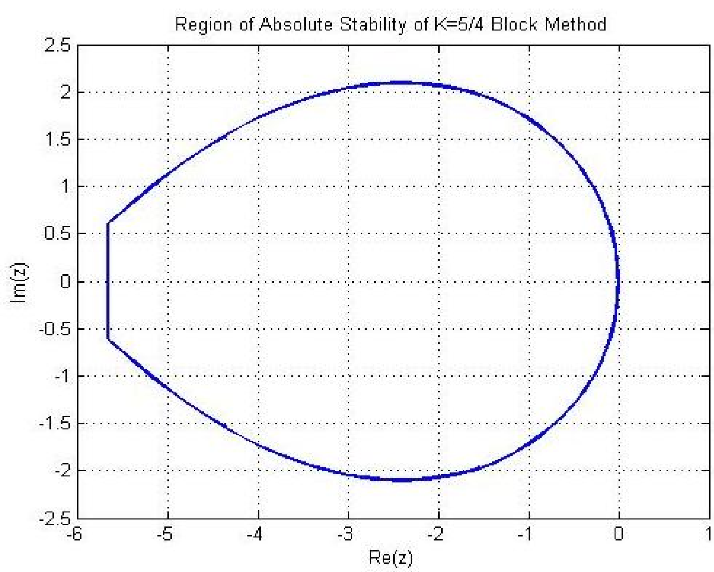

Fig. 1. It shows the region of absolute stability of the developed method

5 Implementation of the Developed Method

In this section, The developed methods is tested on some sample of initial value problem of second order ordinary differential equation. The problem solved ranges from linear, non linear, oscillatory and application problem (Dynamics). The absolute error of problem IV is compared with Ali Shokri 2014. All the methods are of the same order $p = 5$.

5.1 Numerical experiments

5.1.1 Problem I: Considered a non-linear second order ordinary differential equations

$$y'' = x(y')^2, \quad y(0) = 1,\ y'(0) = \frac{1}{2},\ h = 0.1$$

Exact solution: $y(x) = 1 + \frac{1}{2}\log\left(\frac{2+x}{2-x}\right)$

5.1.2 Problem II: considered a highly oscillatory test problem

$$y'' + y' = 0,\ \ y(0) = 1,\ \ y'(0) = 2,\ \ h = 0.01$$

Exact solution $y(x) = \cos(2x) + \sin(2x)$

5.1.3 Problem III: consider a non-linear second order ordinary differential equations

$$y'' = \frac{(y')^2}{2y} - 2y, \quad y(pi/6) = 1,\ y'(pi/6) = \frac{1}{2},\ h = 1/320$$

Exact solution: $y(x) = \sin(x)\,{}^\wedge 2$

5.1.4 Problem IV: Consider a non-linear second order ordinary differential equations

$$y'' = 50y^3,\ \ y(1) = 1/6,\ \ y'(1) = -5/36,\ \ \ h = 0.1, h = 0.01, h = 0.001$$

Exact solution $y(x) = 1/(1+5x)$

5.1.5 Problem V: Considered a non-linear second order ordinary differential equations

$$y'' + \left(\frac{6}{x}\right)y' + \left(\frac{4}{x^2}\right)y = 0, y(1) = 1,\ y'(1) = 1,\ h = \frac{0.1}{32}$$

Exact solution: $y(x) = \frac{5}{3x} - \frac{2}{3x^4}$

5.1.6 Problem VI: Considered a linear second order ordinary differential equations

$$y'' = -y', \; y(0)=1, \; y'(0)=1, \quad h=0.1$$

Exact solution $y(x)=\cos(x)+\sin(x)$

5.1.7 Problem VII: Considered a problem in Engineering 'Dynamic Problem' solved by Areo and Omojola [12].

A 10kg mass is attached to a spring having a spring constant of 140N/m. The mass is started in motion from the equilibrium position with an initial velocity of 1m/s in the upward direction and with an applied external force $F(t)=5\sin t$. Find the subsequent motion of the mass if the force due to air resistance is $-90xN$.

To obtained the exact solution $x(t)=-\frac{9}{50}e^{-2t}+\frac{99}{500}e^{-7t}-\frac{9}{500}\cos(t)+\frac{13}{500}\sin(t)$

It follows that, the exponential terms, which come from the homogenous solution represent an associated free over damped motion, quickly die out. These terms are the transient part of the solution. The terms coming from the particular solution, however, do not die out at t tends to infinity. They are the steady-state part of the solution.

5.2 Table of results

Table 1. Result of test problem 1

X-values	Y-exact	Y-computed	Error in our method	Executed time
0.0009766	1.000488281288805200	1.000488281288806300	1.11022302e-15	**0.0034**
0.0019531	1.000976562810441000	1.000976562810443700	2.66453526e-15	**0.0064**
0.0029297	1.001464844797739200	1.001464844797744300	5.10702591e-15	**0.0075**
0.0039063	1.001953127483532700	1.001953127483549800	1.70974346e-14	**0.0082**
0.0048828	1.002441411100655700	1.002441411100682300	2.66453526e-14	**0.0089**
0.0058594	1.002929695881946500	1.002929695881987300	4.08562073e-14	**0.0093**
0.0068359	1.003417982060245100	1.003417982060305500	6.03961325e-14	**0.0096**
0.0078125	1.003906269868396700	1.003906269868545700	1.48991930e-13	**0.0101**
0.0087891	1.004394559539250900	1.004394559539424800	1.73860926e-13	**0.0105**
0.0097656	1.004882851305662500	1.004882851305876500	2.14050999e-13	**0.0109**

Table 2. Result of test problem 2

X-values	Y-exact	Y-computed	Error in our method	Executed time
0.1000000	1.178735908636302700	1.178735908636302300	4.44089210e-16	0.0165
0.2000000	1.310479336311535700	1.310479336311531900	3.77475828e-15	0.0281
0.3000000	1.389978088304713700	1.389978088304704600	9.10382880e-15	0.0388
0.4000000	1.414062800246688200	1.414062800246671100	1.70974346e-14	0.0499
0.5000000	1.381773290676036000	1.381773290676009200	2.68673972e-14	0.0612
0.6000000	1.294396840443900100	1.294396840443861500	3.86357613e-14	0.0708
0.7000000	1.155416872888702000	1.155416872888650500	5.15143483e-14	0.1275
0.8000000	0.970374080740218140	0.970374080740153080	6.50590692e-14	0.1395
0.9000000	0.746645536185110980	0.746645536185032710	7.82707232e-14	0.1488
1.0000000	0.493150590278543470	0.493150590278453260	9.02056208e-14	0.1589

Table 3. Result of Test Problem 3

X-values	Y-exact	Y-computed	Error in our method	Executed time
0.5245753	0.250846204232690550	0.250846207686758300	3.45406775e-09	0.0465
0.5255519	0.251693358911382450	0.251693368177742520	9.26636007e-09	0.0538
0.5265285	0.252541460804438120	0.252541477525292320	1.67208542e-08	0.0624
0.5275050	0.253390506676606510	0.253390554442725680	4.77661192e-08	0.0707
0.5284816	0.254240493289035530	0.254240544431130700	5.11420952e-08	0.0742
0.5294582	0.255091417399285040	0.255091474222082560	5.68227975e-08	0.0757
0.5304347	0.255943275761337740	0.255943339869503130	6.41081654e-08	0.0762
0.5314113	0.256796065125613200	0.256796159576889450	9.44512762e-08	0.0765
0.5323878	0.257649782238979000	0.257649879987756620	9.77487776e-08	0.0788
0.5333644	0.258504423844763960	0.258504527142997990	1.03298234e-07	0.0806

Table 4. Result of test problem 4, when h=0.1, 0.01 and 0.001

X-values	Y-Exact	Y-Computed	Error in our method	Error in Ali Shokri [14]
5.1250000	0.037558685446009391	0.037558361726772745	3.23719237e-07	1.1325e-05
10.0625000	0.019488428745432398	0.019486108175352040	2.32057008e-06	4.2034e-05
15.0625000	0.013104013104013105	0.013096380020518012	7.63308350e-06	6.1478e-04
20.0625000	0.009870450339296731	0.009852593037001587	1.78573023e-05	9.0336e-04
25.0625000	0.007916872835230085	0.007882280414309299	3.45924209e-05	-
		When h=0.01		
X-values	**Y-Exact**	**Y-Computed**	**Error in our method**	**Error in Ali shokri [14]**
5.0500000	0.038095238095237807	0.038095238094946401	2.91405788e-13	2.7745e-07
10.0500000	0.019512195121951285	0.019512195119777535	2.17375076e-12	4.0120e-07
15.0500000	0.013114754098360930	0.013114754091201662	7.15926797e-12	5.1436e-07
20.0500000	0.009876543209876838	0.009876543193114824	1.67620137e-11	8.1129e-06
25.0500000	0.007920792079208198	0.007920792046712068	3.24961308e-11	-

	When h=0.001			
X-values	**Y-Exact**	**Y-Computed**	**Error in our Method**	**Error in Ali Shokri [14]**
5.0006250	0.038456916236028642	0.038456916236028524	1.17961196e-16	2.0025e-10
10.0012500	0.019605440509740456	0.019605440509734749	5.70724024e-15	5.8625e-10
15.0006250	0.013157353727228947	0.013157353727209454	1.94930877e-14	6.0369e-10
20.0006250	0.009900683765972612	0.009900683765920538	5.20746640e-14	8.4412e-09
25.0006250	0.007936311103396701	0.007936311103294196	1.02504810e-13	**-**

Table 5. Result of test problem 5

X-values	Y-Exact	Y-Computed	Error in our Method	Executed Time
1.010	1.009299459336012900	1.009301243863692600	1.78452768e-06	**0.1020**
1.012	1.011050479081722700	1.011052894071904800	2.41499018e-06	**0.1054**
1.013	1.011912777775976200	1.011915253507538000	2.47573156e-06	**0.1061**
1.014	1.012766348584467100	1.012768921972228200	2.57338776e-06	**0.1066**
1.015	1.013611250648484200	1.013613947270816800	2.69662233e-06	**0.1070**
1.016	1.014447542686413700	1.014450731722504900	3.18903609e-06	**0.1077**
1.017	1.015275282997085700	1.015278533191526800	3.25019444e-06	**0.1083**
1.018	1.016094529463085900	1.016097876587227300	3.34712414e-06	**0.1087**
1.019	1.016905339554039900	1.016908808389307400	3.46883527e-06	**0.1091**
1.020	1.017707770329870300	1.017711718765947900	3.94843608e-06	**0.1094**

Table 6. Result of test problem 6

X-values	Y-Exact	Y-Computed	Error in our Method	Executed Time
0.001	1.000976085507659300	1.000976085507659300	0.00000000e+00	0.0051
0.002	1.001951216410210200	1.001951216410210200	0.00000000e+00	0.0075
0.003	1.002925391777695900	1.002925391777695900	0.00000000e+00	0.0083
0.004	1.003898610681070100	1.003898610681070100	0.00000000e+00	0.0092
0.005	1.004870872192199100	1.004870872192199100	0.00000000e+00	0.0098
0.006	1.005842175383862200	1.005842175383862200	0.00000000e+00	0.0101
0.007	1.006812519329752600	1.006812519329752400	2.22044605e-16	0.0104
0.008	1.007781903104477900	1.007781903104477900	0.00000000e+00	0.0107
0.009	1.008750325783562200	1.008750325783562200	0.00000000e+00	0.0110
0.010	1.009717786443445700	1.009717786443445700	0.00000000e+00	0.0119

Table 7. Result of test problem 7

X-values	Y-Exact	Y-Computed	Error in our Method	Executed Time
0.0009766	-0.000972281269853562	-0.000972361595065321	8.03252118e-08	**0.0604**
0.0019531	-0.001936041153867146	-0.001936256621243597	2.15467376e-07	**0.0755**
0.0029297	-0.002891340944060578	-0.002891729640479877	3.88696419e-07	**0.0858**
0.0039063	-0.003838241508575083	-0.003839351892888554	1.11038431e-06	**0.0905**
0.0048828	-0.004776803294573667	-0.004777991628559325	1.18833399e-06	**0.0937**
0.0058594	-0.005707086331121749	-0.005708405856426478	1.31952530e-06	**0.0948**
0.0068359	-0.006629150232048403	-0.006630637936799994	1.48770475e-06	**0.0952**
0.0078125	-0.007543054198787732	0.007545242906571024	2.18870778e-06	**0.0955**
0.0087891	-0.008448857023201178	-0.008451121402863710	2.26437966e-06	**0.0961**
0.0097656	-0.009346617090380382	-0.009349008851141006	2.39176076e-06	**0.0967**
0.0107422	-0.010236392381430894	-0.010238947442496262	2.55506107e-06	**0.0972**

6 Conclusion

This research paper proposes a Fully Implicit five-quarters Computational Algorithms of uniform order five for Numerical Approximation of Second order IVPs in ODEs. The method is consistent, convergent, zero stable and A-stable as shown in figure 1 above. The method solves second order ordinary differential equations efficiently and accurately. It has a low error constant and gives better approximation. The method also solves application problem "Dynamic Problem'. The developed method is specifically compared with Ali Shokri (2014). The results are presented in table 1-7 above.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Adesanya AO, Odekunle M, Alkali MA. Three steps block predictor-block corrector method for the solution of general second order ordinary differential equations. International Journal of Engineering Research and Applications (IJERA). 2012;2(4):2297-2301.
[ISSN: 2248-9622]
Available:www.ijera.com

[2] Ogunware BG, Omole EO, Olanegan OO. Hybrid and non-hybrid implicit schemes for solving third order ODEs using block method as predictors. Journal of Mathematical Theory and Modelling. 2015; 5:10-26.

[3] Olabode BT, Omole EO. Implicit hybrid block numerov-type methods for direct solution of fourth order ordinary differential equations. Journal of Computational and Applied Mathematics. 2016;5(5): 129–139.

[4] Odekunle MR, Adesanya AO, Sunday JA. New block integrator for the solution of initial value problems of first order ordinary differential equations. Int. J. pure Applied Science Technology. 2012; 11(1):92–100.

[5] Ibijola EA, Skwame Y, Kumleng G. Formation of hybrid of higher step-size, through the continuous multistep collocation. American Journal of Scientific and Industrial Research. 2011;2:161-173. Available:http://dx.doi.org/10.5251/ajsir.2011.2.2.161.173

[6] Norsett. One step method of hermite type for numerical integration of stiff system. BIT. 1974;14:63–77.

[7] Kayode SJ, Obarhua FO. Continuous y-function hybrid methods for direct solutions of differential equations. International Journal of Differential Equations and Application. 2013;12(1):37–48.

[8] Kuboye JO, Omar Z. Multistep collocation block method for direct solution of second order ordinary differential equations. American journal of applied sciences. 2015;12(9):663-668.

[9] Jator SN. A sixth order linear multistep method for the direct solution of $y'' = (x, y, y')$, International Journal of Pure and Applied Mathematics. 2007a;40:457-472.

[10] Mohammed U, Adeniyi RB. Derivation of five step block hybrid backward differential formulas (HBDF) through the continuous multi-step collocation for solving second order differential equation. Pacific J. Sci. Technol. 2014;15:89-95.

[11] Adoghe LO, Akhigbe J. A comparative study of a class of implicit multiderivative methods for the numerical solution of second-order ordinary differential equations. Journal of Applied Mathematics & Bioinformatics. 2015;5:107-131.

[12] Areo EA, Omojola MT. A new one-twelfth continuous block method for the solution of modelled problems of ordinary differential equations. American Journal of Computational Mathematics. 2015; 5:447-457.

[13] Onumanyi P, Sirisena UW, Jator SA. Solving difference equation. International Journal of Computing Mathematics. 1999;72:15-27.
Available:http://dx.doi.org/10.1080/00207169908804831

[14] Ali Shokri. The symmetric p-stable hybrid obrechkoff methods for the numerical solution of second order ivps, TWMS J. Pure Appl. Math. 2014;5(1):28-35.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle3.com/review-history/44846