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Differential Transform Decomposition Method for the Solution of Boundary Layer Equations in a Finite Domain

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Authors' contributions

This work was carried out in collaboration between all authors. Author RAO formulated the method, applied the method to solve the set of problems considered, and wrote the first draft of the manuscript. Authors FOA and YASA checked the mathematical calculations to make sure they are correct. All authors read and approved the final manuscript.

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Abstract

A new method called differential transform decomposition method (DTDM) for solving differential equations was developed. This method was derived by coupling the scheme of Differential Transform with Adomain polynomials,the necessity of the Adomian polynomial is to decompose the the non linear functions existing in a differential equation so that the differential transform of such functions could be obtained easily.To validate the efficiency of the proposed method, a single and coupled boundary value problems in a finite domain were considered.

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The results obtained were presented in a polynomial form so that the solution at any point of the problems considered could be obtained as against some methods which are discritised. Computational results agree with the referenced solution for the single boundary value problem and for the coupled boundary value problems the results agrees with that of the weighted residual method.

Keywords: Differential transform decomposition method; boundary layer equations; Adomian polynomials.

AMS Subject Classification: 65M60, 65N30

1 Introduction

Fluid dynamics is an important aspect of applies physics and Engineering. The necessity for the developing the understanding of this field is indispensable, for example when one considers the amount of fluid in the surrounding environment, the large quantities of fluid operating in human body on a daily basis. Of particular interest in fluid dynamics is the study of laminar boundary layer flow of an incompressible fluid. Practical examples of boundary layer theory are aerodynamic extraction of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along liquid film condensation process [1].

The pioneering work in this area was done by sakiadis(1961), over years similarity solutions were proposed, for stretching walls and porous medium [2]. Owing to engineering applications of this area, convective flows over wedge shaped bodies have been extensively studied since early formulation of the problem in 1931 by Falkner and Skan[3].However, due to the nonlinearity of the aforementioned physical problems, analytical solutions of such are not easily obtained; hence numerical and approximate analytical solutions are always employed to tackle these sets of problems. Among these methods are homotopy perturbation, Variational iteration and Modified Variational methods, Weighted residual method and differential Transform method, details of these methods could be found in [4], [5],[6], [7] and [8] respectively. The interest of this work is the semi analytic methods for solving the said nonlinear equations. Adomian decomposition method was introduced by George Adomian in the $1980's$ for solving linear and nonlinear functional equations [9], over the years the method has been applied to obtain solutions to a wide class of differential equations: [10] used Adomian decomposition method to solve parabolic equations in an infinite domain. [11] applied Adomian decomposition method on certain singular initial value problems. [8] used the method of differential transform to solve linear and nonlinear initial value problems whose solution were compared with that of Adomian. [12] considered MHD flow and heat transfer of a couple stress fluid over oscillatory stretching sheet embedded in a porous medium in the presence of heat source/sink, in their work homotopy analysis was used to solve the system of equations obtained in order to investigate the effect of some physical parameters. [13] also investigated the thermal-diffusion and diffusion-thermo effects on magnetohydrodynamic viscoelastic flow of second grade fluid over a porous oscillatory stretching sheet with thermal radiation where the dimensionless nonlinear partial differential equations were solved by Homotopy analysis method.

In this letter, Differential transform Decomposition Method(DTDM) which is obtained through Differential Transform Scheme and Adomian polynomial is used to obtain semi analytical solution of boundary layer equations in a finite domain.This is done to avoid the repeated integration in Adomian Decomposition method and to decompose the nonlinear terms so that the differential transform the resulting functions could easily be otained

2 Method of Solution

Suppose we have differential equation of the form [11]

$$Lu + Ru + Nu = g(t) \tag{2.1}$$

where L is the operation of the highest ordered derivative or the space variable and R is the remainder of the linear operator, N(u) being the nonlinear term and g(t) the forcing term. In differential Transform method, to solve equation 2.1 we transform as

$$D^T(Lu) + D^T(Ru) + D^T(Nu) = D^T(g(t)) \tag{2.2}$$

before iterating.

But with the proposed method equation (2.1) will transformed as

$$D^T(Lu) + D^T(Ru) + A_n = D^T(g(t))$$

where A_n is the Adomian decomposition of $N(u)$, D^T is the corresponding transform of each term and the nonlinear term decomposed by using Adomian polynomials which is given by George Adomian.

Adomian[9] gave a polynomial of the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(N\sum_{r=0}^{\infty}\lambda^r u_r)|_{\lambda=0} \quad n = 0, 1, 2, ...$$

for decomposing the nonlinear term in any differential equation. Of course, we can infer modifications of A_n for products of different dependent fuctions as

$$N(U, V, W) = UVW$$

in the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(N\sum_{r=0}^{\infty}\lambda^r U_r).(N\sum_{r=0}^{\infty}\lambda^r V_r).(N\sum_{r=0}^{\infty}\lambda^r W_r)|_{\lambda=0} \quad n = 0, 1, 2, ...$$

Definition 2.1[8]:The transformation of the kth derivative of a function in one variable is as

$$F_k = \frac{1}{k!}[\frac{d^k f(t)}{dt^k}]_{t=t_0}$$

and the inverse transformation is

$$f(t) = \sum_{k=0}^{\infty} F_k(t - t_0)^k$$

where F_k is the differential transform of $f(t)$. Table 2.1 below shows the transform of functions from the basic definitions of differential transform as provided by [8].

Table 2.1

Functional form	Transformed form
$f(t) = u(t) \pm v(t)$	$F_k = U_k \pm V_k$
$f(t) = \lambda u(t)$	$F_k = \lambda U_k$
$f(t) = \frac{d^n u(t)}{dt^n}$	$F_k = \frac{(k+1)!}{k!} U(k+n)$
$f(t) = t\frac{du(t)}{dt}$	$U_k = \sum_{r=0}^{k} \delta(r-1)(k-r+1)U_{k-r+1}$
$f(t) = t\frac{d^2u(t)}{dt^2}$	$U_k = \sum_{r=0}^{k} \delta(r-1)(k-r+1)(k-r+2)U_{k-r+2}$
$f(t) = \frac{du(t)}{dt}\frac{du(t)}{dt}$	$U_k = \sum_{r=0}^{k}(r+1)(k-r+1)U_{r+1}U_{k-r+1}$
$f(t) = \frac{d^2u(t)}{dt^2}\frac{d^2u(t)}{dt^2}$	$U_k = \sum_{r=0}^{k}(r+1)(r+2)(k-r+1)(k-r+2)U_{r+2}U_{k-r+2}$
$f(t) = u(t)\frac{d^2u(t)}{dt^2}$	$U_k = \sum_{r=0}^{k}(k-r+1)(k-r+2)U_rU_{k-r+2}$

2.1 Description of differential transform decomposition method

Consider the third order non-homogeneous nonlinear ordinary differential equation with initial conditions given by

$$f''' + b_1 f f'' + b_2 f'^2 + b_3 f = 0 \tag{2.3}$$

$$f(0) = \alpha, \quad f'(0) = \beta, \quad f''(0) = \gamma \tag{2.4}$$

where f', f'' and f''' are the derivatives of f with respect to η and b_1, b_2 b_3,β, γ are constants. The nonlinear terms in equation (2.3) are

$$A = f'' \quad and \quad B = f'^2 \tag{2.5}$$

substituting equation (2.5) into equation (2.1), we have

$$f''' + b_1 A + b_2 B + b_3 f = 0 \tag{2.6}$$

Now using the differential transform in table 2.1 we have

$$f_{k+3} = \frac{-1}{(k+1)(k+2)(k+3)}(b_1 A_k + b_2 B_k + b_3 f_k) \quad k = 0, 1, 2, 3... \tag{2.7}$$

from the initial condition (2.4)

$$f(0) = \alpha \;\; implies \;\; f_0 = \alpha \quad f'(0) = \beta \;\; implies \;\; (k+1)f_{k+1} = \beta \;\; f_1 = \beta \tag{2.8}$$

$$f''(0) = \gamma \quad (k+1)(k+2)f_{k+2} = \gamma \quad f_2 = \frac{\gamma}{2} \tag{2.9}$$

where A_k and B_k are the decomposed expressions of ff'' and f'^2 respectively using Adomian polynomial. where

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\{(\sum_{r=0}^{n}\lambda^r f_r)(\sum_{r=0}^{n}\lambda^r (r+1)(r+2)f_{r+2})\}|_{\lambda=0} \;\; n = 0, 1, 2... \tag{2.10}$$

and

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}((\sum_{r=0}^{n}\lambda^r (r+1)f_{r+1})(\sum_{r=0}^{n}\lambda^r (r+1)f_{r+1}))|_{\lambda=0} \;\; n = 0, 1, 2... \tag{2.11}$$

Solving equations (2.7,2.8,2.9,2.10 and 2.11) simultaneously to obtain the iterative terms f_3,f_4,f_5... and finally the overall solution using the inverse transform formula in definition 2. 1 as

$$f(\eta) = \sum_{i=0}^{n} f_i \eta^i \quad n \quad is \quad fixed \quad number \tag{2.12}$$

2.2 Summary of the steps involves in using differential transform decomposition method

1. Identify the nonlinear term in the differential equation given.
2. Apply Adomian polynomial to decompose the identified nonlinear term(s)
3. Apply differential transform to each term of the differential equation including the decomposed function(s).
4. Find the required iterative steps and sum them up.
5. Find the inverse differential transform of the summation obtained in step 4.

3 Numerical Examples

The fluid problem given by [14] which is govern by the differential equation

$$f^{(iv)} + S(-\eta f''' - 3f'' - \beta f'f'' + ff''') = 0 \tag{3.1}$$

with the boundary conditions

$$f(0) = 0, \;\; f''(0) = 0, \;\; f(1) = 1, \;\; f'(1) = 0 \tag{3.2}$$

which describes two dimensional and one dimensional(Axisymmetric) unsteady flows due to normally expanding or contracting parallel plates. That is

$$\beta = \begin{cases} 0 & Axisymmetric\ case \\ 1 & two\ dimensional \end{cases} \tag{3.3}$$

S is the squeeze number which is a non dimensional parameter that characterized the flow

Applying the differential transform decomposition method discussed in section (2.1)to solve equation (3.1) subject to the boundary condition 3.2 , equation 3.1 will be transformed as $f_{k+4} = \frac{S}{(k+1)(k+2)(k+3)(k+4)}(\sum_{r=0}^{k} \delta_{r-1}(k-r+1)(k-r+2)(k-r+3)f_{k-r+3} + 3(k+1)(k+2)f_{k+2} + \beta A_k - B_k)\;\; k = 0, 1, 2, ...(3.4)$
Where

$$A_k = f'f'' = (k+1)f_{k+1}(k+1)(k+2)f_{k+2}$$

and

$$B_k = ff''' = f_k(k+1)(k+2)(k+3)f_{k+3}$$

A_k and B_k are decomposed by Adomian polynomials as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}((\sum_{i=0}^{n} \lambda^i (i+1)f_{i+1})(\sum_{i=0}^{n} \lambda^i (i+1)(i+2)f_{i+2}))|_{\lambda=0}$$

and

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}((\sum_{i=0}^{n} \lambda^i f_i)(\sum_{i=0}^{n} \lambda^i (i+1)(i+2)(i+3)f_{i+3}))|_{\lambda=0}$$

from the boundary conditions we have that $f(0) = 0$ implies $f_0 = 0$

$f''(0) = 0$ which implies $f_2 = 0$

we need to set $f'(0) = \alpha$ and $f'''(0) = b$ where α and b are to be determined. This implies that $f_1 = \alpha$ and $f_3 = \frac{b}{6}$

By decomposing the polynomials A_n and B_n we have

$$A_0 = 2f_1f_2 \quad B_0 = 6f_0f_3$$

$$A_1 = \alpha b \quad B_1 = \alpha b$$
$$A_2 = 12\alpha f_4 \quad B_2 = 24\alpha f_4$$
$$A_3 = \frac{1}{2}b^2 + 20\alpha f_5 \quad B_3 = \frac{1}{6}b^2 + 60\alpha f_5$$
$$A_4 = 10f_4b + 30\alpha f_6 \quad B_4 = 5f_4b + 120\alpha f_6$$
$$A_5 = 15f_5b + 48f_4^2 + 42\alpha f_7 \quad B_5 = 11f_5b + 24f_4^2 + 210\alpha f_7$$

Substituting the polynomial A_i and B_i into equation (3.4) and iterating the resulting expression, this gives

$$f_4 = 0$$
$$f_5 \frac{1}{120}Sb(4 + \beta\alpha - \alpha)$$
$$f_6 = 0$$
$$f_7 = \frac{1}{5040}Sb(24S + 10S\beta\alpha - 18S\alpha + 3\beta b + S\alpha^2\beta^2 - 4S\alpha^2\beta - b + 3S\alpha^2)$$
$$f_8 = 0$$

$f_9 = \frac{1}{362880}S^2b(-264S\alpha + 84\beta b + 114S\alpha^2 + 16\alpha b - 15S\alpha^3 + 104S\beta\alpha + 18S\alpha^2\beta^2 - 100S\alpha^2\beta - 42\beta\alpha b - 9S\alpha^3\beta^2 + 23S\alpha^3\beta^2\alpha + S\alpha^3\beta^3 + 192S - 52b)$

Now Applying the inverse transform on $f_0 ... f_9$ to obtain the solution as

$$f = \sum_{i=0}^{9} f_i\eta^i \tag{3.5}$$

To obtain the values of α and b in the resulting equation (3.6), the two boundary conditions left over, that is the boundary conditions at the other end $f(1) = 1$ and $f'(1) = 0$ will be imposed on the resulting equation (3.5). This procedure yield two simultaneous equations in b and α which can be solved to obtain the values of the constants in each case depending on the value of β and S. The obtained values of α and b corresponds to $f'(0)$ and $f'''(0)$ respectively which on substtution into equation (3.5) gives the general solution.

Tables 3.1 and 3.2 shows the results of $f(\eta)$ for both positive and negative S in comparison with that obtained through Runge-Kutta method of order four[14].

Table 3.1: Showing the results of equation (3.1) for axisymmetric case $\beta = 0$ in comparison with the results of Runge-Kutta method of order four

S	η	$f(\eta)$ proposed	R-k method [14]
-1.5	0.2	0.319289	0.319526
	0.4	0.603438	0.603830
	0.6	0.822480	0.822876
	0.8	0.956588	0.956801
-0.5	0.2	0.302580	0.302582
	0.4	0.578079	0.578082
	0.6	0.800777	0.800780
	0.8	0.947700	0.947702
0.5	0.2	0.290330	0.290322
	0.4	0.559266	0.559252
	0.6	0.784318	0.784303
	0.8	0.940711	0.940703
1.5	0.2	0.281110	0.281010
	0.4	0.544952	0.544779
	0.6	0.771556	0.771371
	0.8	0.935142	0.935036

Table 3.2: Showing the results of equation (3.1) for axisymmetric case $\beta = 1$ in comparison with the results of Runge-Kutta method of order four

S	η	$f(\eta)$ proposed	[14]
-1.5	0.2	0.333846	0.333618
	0.4	0.624728	0.624358
	0.6	0.839687	0.839325
	0.8	0.963169	0.962984
-0.5	0.2	0.305546	0.305545
	0.4	0.582472	0.582470
	0.6	0.804394	0.804382
	0.8	0.949109	0.949108
0.5	0.2	0.288256	0.288260
	0.4	0.556137	0.556143
	0.6	0.781664	0.781671
	0.8	0.939636	0.939640
1.5	0.2	0.276407	0.276432
	0.4	0.537709	0.537752
	0.6	0.765202	0.765249
	0.8	0.932444	0.932471

The above example is for single boundary layer equation in a finite domain, this procedure is also efficient for the solution of coupled boundary value problems. Now considering the coupled boundary layer equations modelled by Ravikumar et al [15], which is governed by

$$-\frac{du}{dy} = \frac{1}{Re}\frac{d^2u}{dy^2} + G_r\theta + G_m\phi - \frac{u}{Re\alpha} - MReu$$

$$-\frac{d\theta}{dy} = \frac{1}{ReP_r}\frac{d^2\theta}{dy^2} + \frac{E}{Re}(\frac{du}{dy})^2$$

$$-\frac{d\phi}{dy} = \frac{1}{ReSc}\frac{d^2\phi}{dy^2} - KRe\phi + S_0\frac{d^2\theta}{dy^2}$$
$$u(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1$$
$$u(1) = 0, \quad \theta(1) = m, \phi(1) = n \tag{3.6}$$

Where P_r is the Prandtl number, Sc is the Schmidt number, Re is the Reynolds number, G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, E is the Eckert number, α is the permeability parameter, M is the Hatmann number, θ is the dimensionless temperature, ϕ is the dimensionless concentration, S_0 is the Soret number, m and n are constants. To solve equation (3.6) using the method discussed above, equation (3.6) will be transformed as

$$U_{s+2} = \frac{Re}{(s+1)(s+2)}\left(\frac{U_s}{Re\alpha} + MReU_s - G_r\theta_r - G_m\phi_s - (s+1)U_{s+1}\right)$$
$$\theta_{s+2} = \frac{Re}{(s+1)(s+2)}\left((s+1)\theta_{s+1} + \frac{E}{Re}A_s\right)$$
$$\phi_{s+2} = \frac{ReSc}{(s+1)(s+2)}(kRe\phi_s - S_0(s+1)(s+2)\theta_{s+2} - (s+1)\phi_{s+1}) \quad s = 0, 1, 2... \tag{3.7}$$

In equation (3.7) the only nonlinear term is $(\frac{du}{dy})^2$ and this is represented with A_s. The expression for $A_s = ((i+1)U_{i+1})^2, \quad i = 0, 1, 2..$ will be generated by Adomian Polynomial as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left(\left(\sum_{i=0}^{n}\lambda^i(i+1)U_{i+1}\right)^2\right)_{\lambda=0} \quad n = 0, 1, 2, ..$$

simplifying $A_n \;\; n = 0, 1, 2...$ then we have

$$A_0 = U_0^2$$
$$A_1 = 4U_1U_2$$
$$A_2 = 4U_2^2 + 6U_1U_3$$
$$A_3 = 12U_2U_3 + 8U_1U_4$$
$$A_4 = 9U_3^2 + 16U_2U_4 + 10U_1U_5$$
$$A_5 = 24U_3U_4 + 20U_2U_5 + 12U_1U_6 \tag{3.8}$$

The boundary conditions $u(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1$ implies that $U_0 = 0, \ \theta_0 = 1, \ \phi_0 = 1$. Since we are dealing with three systems of ordinary differential equations then there is a need for three additional conditions at the initial point, that which are assumed to be $u'(0) = c, \ \ \theta'(0) = e, \ \ \phi'(0) = f$, transforming these assumed equations gives $U_1 = c, \theta_1 = e, \phi_1 = f$ with these constants to be determined later. Solving equations (3.7) and (3.8) simultaneously, then we iteratively obtain $U_s, \theta_s, \phi_s \;\; s = 0, 1, 2..$

$$U_2 = \frac{1}{2}R\left(\frac{U_0}{R\alpha} + G\theta_0 - G_r\phi_0 - U_1\right)$$
$$\theta_2 = \frac{1}{2}RP_r\left(\theta_1 + \frac{EA_0}{R}\right)$$
$$\phi_2 = \frac{1}{2}RS_c\left(kR\phi_0 - S_0RP_r\left(\theta_1 + \frac{EA_0}{R}\right) - \phi_1\right) \tag{3.9}$$

. . . So that the solution of equation (3.6) becomes

$$u(y) = \sum_{i=0}^{n} U_i y^i$$

$$\theta(y) = \sum_{i=0}^{n} \theta_i y^i$$

$$\phi(y) = \sum_{i=0}^{n} \phi_i y^i \qquad (3.10)$$

Equations (3.10) give the partial solution because it still contains some assumed constants that are yet to be determined. In order to obtain these constants, we impose the left over boundary conditions in equation (3.6), that is

$$u(1) = 0, \quad \theta(1) = m, \phi(1) = n$$

into equation (3.10) and these give three equations in c, e and f which are solved simultaneously when $m = n = 2$ and $M = Re = \alpha = 1$ $Pr = 0.71$, $Sc = k = 0.5$, $G_r = G_m = 5$, $S_0 = 2.5$, $E = 0.01$ to obtain

$$c = 6.70890773, \quad e = 0.7829362318, \quad f = 1.375214171$$

substituting the calculated constants into equation 3.10 gives

$u(y) = 6.708907773y - 8.354453886y^2 + 3.222661884y^3 - 2.077953980y^4 + 0.6874688490y^5 - 0.234181473y^6 + 0.05806468498y^7 - 0.01207117966y^8 + 0.1544185067 * 10^{-2}y^9 + 0.1018991820 * 10^{-3}y^{10} - 0.1587470222 * 10^{-3}y^{11} + 0.6998911775 * 10^{-4}y^{12}$ (3.11)

$\theta(y) = 1 + 1.014139121y + 0.391693880y^2 - 1.391626123y^3 + 1.706853878y^4 - 1.906168996y^5 + 1.890937632y^6 - 1.607026349y^7 + 1.260956639y^8 - 0.8864051162y^9 + 0.5836058052y^{10} - 0.358766127y^{11} + 0.2115302496y^{12}$ (3.12)

$\phi(y) = 1 + 0.9514981261y - 0.2457648245y^2 + 0.6029344325y^3 - 0.4778351335y^4 + 0.3114865901y^5 - 0.2247770974y^6 + 0.1357278591y^7 - 0.07971112440y^8 + 0.04372175835y^9 - 0.02322012250y^{10} + 0.01174245372y^{11} - 0.005802916773y^{12}$ (3.13)

equations (3.11) ,(3.12) and (3.13) are the general solutions of coupled equation (3.6).

Table 3.3: Showing the results of $u(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	0.0000	0.0000	0.0000
0.1	0.5891806455	0.5903677511	0.0011871056
0.2	1.0281041850	1.0302656840	0.0021614990
0.3	1.3296381570	1.3324636950	0.0028255380
0.4	1.5029480360	1.5060733320	0.0031252960
0.5	1.5539657790	1.5570348740	0.0030690950
0.6	1.4857870260	1.4885034480	0.0027164220
0.7	1.2989969730	1.3011508040	0.0021530310
0.8	0.9919248944	0.9933904004	0.0014655060
0.9	0.5608273421	0.5615537402	0.0007263981
1.0	0.0000	0.0000	0.0000

Table 3.4: Showing the results of $\theta(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	1.0000	1.0000	1.0000
0.1	1.1034339870	1.1130953450	0.0096613580
0.2	1.2049619360	1.2275978610	0.0226359250
0.3	1.3054941450	1.3392338440	0.0337396990
0.4	1.4056113110	1.4452390070	0.0396276960
0.5	1.5056255470	1.5438509790	0.0382254320
0.6	1.6056224990	1.6341569160	0.0285344170
0.7	1.7054846470	1.7164430990	0.0109584520
0.8	1.8048957040	1.7938179300	0.0110777740
0.9	1.9033261690	1.8773416550	0.0259845140
1.0	2.0000	2.0000	0.0000

Table 3.5: Showing the results of $\phi(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	1.0000	1.0000	1.0000
0.1	1.1064212120	1.0932502180	0.0131709940
0.2	1.2108152490	1.1846148140	0.0262004350
0.3	1.3133135630	1.2763576140	0.0369559490
0.4	1.4141168120	1.3700805990	0.0440362130
0.5	1.5134775770	1.4668478160	0.0466297610
0.6	1.6116881240	1.5672547910	0.0444333330
0.7	1.7090731900	1.6714518400	0.0376213500
0.8	1.8059878310	1.7791099060	0.0268779250
0.9	1.9028203090	1.8892778570	0.0135424520
1.0	2.0000	2.0000	0.0000

4 Results and Discussion

Tables 3.1 and 3.2 show the results of equation 3.1 subject to the boundary condition 3.2 for both axisymmetric and two dimensional cases. The results obtained were compared with that of Saeed and Moradi (2012) who used Runge-Kutta method for the solution of the problem. From the tables it was observed that the results of the proposed method that is ($DTDM$) is in good agreement with that of Runge-Kutta method of order four obtained by Saeed and Moradi (2012), for different values of the squeeze parameter S considered.

Also tables 3.3 to 3.5 show the solution of velocity, temperature and concentration of equation (3.6). For comparison purpose, this results were compared with the solution obtained while method of weighted residual was used. The difference between the solutions of the two methods were negligible and this affirms the efficiency of the proposed method. However, the proposed method presents the results in semi analytical form which is not in discritized form; this shows the advantage of the proposed method over Runge Kutta method.

5 Conclusion

The method of differential transform decomposition has been employed to solve fluid dynamics problem in a finite domain for both single and coupled equations.

This method solve this class of differential equation without any need of discritization of the variables. It small size of computation in comparison with the computational size required in some other numerical methods show that the method is reliable and introduces a significant improvement in solving physical phenomena.

Competing Interests

Authors have declared that no competing interests exist.

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