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Dynamic Response to Variable-magnitude Moving Distributed Masses of Bernoulli-Euler Beam Resting on Bi-parametric Elastic Foundation

Akintomide Adeniyi [1*] and Awodola Thomas Olubunmi [1]

[1] *Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria.*

Authors' contributions

This work was carried out in collaboration between both authors. Author AA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author ATO managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

This work investigates the problem of dynamic response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on bi-parametric elastic foundation. The governing equation is a fourth order partial differential equation with variable and singular co-efficients. This equation is reduced to a set of coupled second order ordinary differential equation by the method of Garlerkin. For the solutions of these equations, two cases are considered; (1) the moving force case – when the inertia is neglected and (2) the moving mass case – when the inertia term is retained. To solve the moving force problem, the Laplace transformation and convolution theory are used to obtain the transverse-displacement response to a moving variable-magnitude distributed force of the Bernoulli-Euler beam resting on a bi-parametric elastic foundation. For the solution of the moving mass problem, the celebrated struble's technique could not simplify the coupled second order ordinary differential equation with singular and variable co-efficient because of the variability of the load magnitude; hence use is made of a numerical technique, precisely the Runge-Kutta of fourth order is used to solve the moving mass problem of the response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on Pasternak elastic foundation. The analytical and the numerical solutions of the moving force problem are

*Corresponding author: E-mail: akintomideadeniyi4@gmail.com;

compared and shown to compare favourably to validate the accuracy of the Runge-Kutta scheme in solving this kind of dynamical problem. The results show that response amplitude of the Bernoulli-Euler beam under variable-magnitude moving load decrease as the axial force N increases for all variants of classical boundary conditions considered. For fixed value of N, the displacements of the beam resting on bi-parametric elastic foundation decrease as the foundation modulus K_0 increases. Furthermore, as the shear modulus G_0 increases, the transverse deflections of the beam decrease. The deflection of moving mass is greater than that of moving force for all the variants of boundary conditions considered, therefore, the moving force solution is not a safe approximation to the moving mass problem. Hence safety is not guaranteed for a design based on the moving force solution for the beam under variable-magnitude moving distributed masses and resting on bi-parametric elastic foundation.

Keywords: Beam; Bernoulli-Euler; Pasternak; Runge-Kutta; axial force; shear modulus.

1 Introduction

The dynamic effects of a load on beam and beam-like structural members play significant role in railway tracks, road tracks and highway pavement designs. In modern engineering practices, beam- like structures resting on both variable and constant elastic foundation have wide applications and for this reason several authors have investigated the dynamic deflection of beam [1-4]. Many structures are designed to support moving masses such as bridges, guide ways, overhead cranes, rails, roadways tunnels, and pipeline etc. Also many structural members can be modeled as beams under moving loads in the design of machining processes. The dynamics responses of a beam acted upon by moving masses have been studied extensively in connection with the design of railway tracks and machining processes by Lee [5]. The equation of motion in matrix form has been formulated for the dynamics response of a beam acted upon by a moving mass by using Lagrangian approach and the assumed mode method, and found that separation of the mass from the beam may occur for a relatively slow speed and small mass when the beam is clamped at both ends.

When the Chester rail bridge collapsed in England, various kinds of moving load problem associated with structural dynamics have been presented in excellent monograph by Fryba [2]. In the Fryba book detailed solution of the problem of a constant force moving along infinite beam over an elastic foundation including its all possible speed and values of viscous damping is presented. Dynamic problem of a simply supported beam subjected to a constant force moving at a constant speed is analyzed by Olsson [6]. Analytical and finite element solutions to this fundamental moving load problem is shown and the result given by the author and other investigators are intended to give a basic understanding of the moving load problem and some computational algorithms discussed. Furthermore, Kenny, [1] took up the problem of investigating the dynamic response of infinite elastic beam on elastic foundation when the beam is under the influence of a dynamic load moving with constant speed. He included the effect of viscous damping in the governing differential equation of motion. Eisenberger and Clastornik [7] solved the problem of a beam on a bi-parametric elastic foundation and presented a finite element procedure for analyzing the flexural vibrations. Cao and Zhong [8] solved the problem of a beam on a Pasternak foundation and under a moving load. The method of Fourier integral is used to obtain the solution to the formulation of the problem. The effect of the moving load velocity on the dynamic displacement response of the beam is discussed. Recently, Awodola [9] considered the influence of foundation and axial force on the vibration of thin beam under variable harmonic moving load. The technique is based on the finite Fourier sine transformation. More recently, Oni and Awodola [10] investigated the dynamic behaviour under moving concentrated masses of simply supported rectangular plates resting on variable Winkler elastic foundation.

Over the years, the problem of assessing the dynamic behavior of structural members subjected to moving loads investigated by several researchers are limited to the case in which the loads are simplified as a harmonic time variable concentrated moving force and cases in which the elastic foundation has been usually modeled by a Winkler foundation [11-14]. This kind of model proposed by Winkler, consisting of a system of mutually independent linear springs, is assumed that the deflection of foundation at any point on the surface is directly proportional to the stress and there is no interaction between the lateral springs. It does not accurately represent the continuous element of many practical foundations. Therefore, to find a more physically close and practically applicable model, bi-parametric foundations were proposed by the following researchers [15-19]. This research work considers bi-parametric foundation model because of the discontinuity of Winkler foundation model. Hence, this research work investigates the dynamic response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on Pasternak elastic foundation.

2 Governing Equation

The problem of assessing the dynamic response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on bi-parametric elastic foundation is considered.

Consider the dynamic response to variable-magnitude moving distributed masses of Bernoulli- Euler beam, resting on bi-parametric elastic foundation, the governing equation of motion is given by the fourth order partial differential equation [15].

$$\frac{\partial^2}{\partial x^2}\left[EJ\frac{\partial^2}{\partial x^2}W(x,t)\right] - \mathrm{N}\frac{\partial^2}{\partial x^2}W(x,t) + \mu\frac{\partial^2}{\partial t^2}W(x,t) + K_0 W(x,t) - G_0\frac{\partial^2}{\partial x^2}\mathrm{W}(x,\mathrm{t}) = \mathrm{P}(x,t) \quad (1)$$

Where, $x$is the spatial co-ordinate, t is the time coordinate, $W(x,t)$is the transverse displacement, E is the Young modulus, J is the moment of inertia, N is the axial force, μ is the mass per unit length of the beam, EJ is the flexural rigidity, K_0 is the foundation modulus, G_0 is the shear modulus, $\mathrm{P}(x,t)$ is the variable-magnitude moving uniformly distributed load on the beam.

When the effect of the mass of the moving load on the response of the beam is taken into consideration, the distributed load $\mathrm{P}(x,t)$ takes the form

$$P(x,t) = \mathrm{P_f}(x,\mathrm{t})\left[1 - \frac{1}{\mathrm{g}}\frac{\mathrm{d}}{\mathrm{dx}}\mathrm{W}(x,\mathrm{t})\right] \quad (2)$$

where,

$\mathrm{P_f}(x,\mathrm{t})$ is the continuous moving force acting on the beam model given by

$$\mathrm{p_f}(x,\mathrm{t}) = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{M_i g}\cos\omega\mathrm{t}\mathrm{H}(x - \mathrm{c_i t}) \quad (3)$$

g is the acceleration due to gravity,

$\frac{\mathrm{d}}{\mathrm{dx}}$ is the convective acceleration defined by Fryba [7]

$$\frac{d}{dx} = \frac{\partial}{\partial \mathrm{t}^2} + 2\mathrm{c}\frac{\partial^2}{\partial x\,\partial \mathrm{t}} + \mathrm{c}^2\frac{\partial^2}{\partial x^2} \quad (4)$$

and $\cos\omega t$ is the variable-magnitude of the moving load.

In this work, the moving load is assumed to move with constant speed c.

Substituting equations (2), (3) and (4) into (1), one obtains

$$\frac{\partial^2}{\partial x^2}\left[EJ\frac{\partial^2}{\partial x^2}W(x,t)\right] - N\frac{\partial^2}{\partial x^2}W(x,t) + \mu\frac{\partial^2}{\partial t^2}W(x,t) + \mathrm{K_0}W(x,t) - \mathrm{G_0}\frac{\partial^2}{\partial x^2}W(x,t) = \sum_{i=1}^{N}\mathrm{M_i g \cos\omega t H(x - c_i t)}\left[1 - \frac{1}{g}\frac{d}{dx}\mathrm{W(x,t)}\right] \tag{5}$$

Equation (6) can also be re-written as

$$\frac{\partial^2}{\partial x^2}\left[EJ\frac{\partial^2}{\partial x^2}W(x,t)\right] - N\frac{\partial^2}{\partial x^2}W(x,t) + \mu\frac{\partial^2}{\partial t^2}W(x,t) + K_0W(x,t) - G_0\frac{\partial^2}{\partial x^2}W(x,t) + \sum_{i=1}^{N}M_i\, g\cos\omega t H(x - c_i t)\left(\frac{\partial^2}{\partial t^2} + 2c_i\frac{\partial^2}{\partial x\partial t} + c_i{}^2\frac{\partial^2}{\partial x^2}\right)w(x,t) = \sum_{i=1}^{N}M_i\, g\cos\omega t H(x - c_i t) \tag{6}$$

The boundary conditions of the structure under consideration are first taken to be arbitrary. The initial condition without any loss of generality is taken as;

$$\mathrm{W}(x,t) = \mathrm{W}(x,0) = 0 = \frac{\partial}{\partial t}W(x,t) = \frac{\partial}{\partial t}W(x,0) \tag{7}$$

3 Analytical Approximate Solution

An exact closed form solution of the above fourth order partial differential equation (1) does not exist. Therefore, an approximate solution is sought. The Galerkin method is employed, this technique requires the solution of equation (1) takes the form

$$W(x,t) = \sum_{m=1}^{n}Y_m\,(t)U_m(x) \tag{8}$$

where,

$$U_m(x) = \sin\frac{\lambda_m x}{L} + A_m\cos\frac{\lambda_m x}{L} + B_m\sinh\frac{\lambda_m x}{L} + C_m\cosh\frac{\lambda_m x}{L} \tag{9}$$

is the beam function chosen so that the concerned boundary conditions are satisfied.

Substituting equation (8) into equation (6), one obtains

$$\sum_{m=1}^{n}\Bigg\{[U_m(x)U_k(x)]\ddot{Y}_m(t) + \frac{EJ}{\mu}[U_m^{IV}(x)U_k(x)]Y_m(t) - \frac{N}{\mu}[U_m^{II}(x)U_k(x)]Y_m(t) + \frac{K_0}{\mu}[U_m(x)U_k(x)]Y_m(t) - \frac{G_0}{\mu}[U_m^{II}(x)U_k(x)]Y_m(t) + \sum_{i=1}^{N}\frac{M}{\mu}[(\cos\omega t H(x-ct)U_m(x)U_k(x))\ddot{Y}_m(t) + 2c(\cos\omega t H(x-ct)U_m(x)U_k(x))\dot{Y}_m(t) + c^2(\cos\omega t H(x-ct)U_m(x)U_k(x))Y_m(t)]\Bigg\}dx = \frac{M_i}{\mu}g\cos\omega t(x-ct)U_k(x)dx \tag{10}$$

In order to determine $Y_m(t)$, it is required that the expression on the left hand side of (10) be orthogonal to the function $U_k(x)$, where k is the dummy index. Therefore, one obtains

$$\sum_{m=1}^{N}\left\{\left(\int_0^L \mathrm{U_m}(\mathrm{x})\mathrm{U_k}\mathrm{dx}\right)\ddot{\mathrm{Y}}_\mathrm{m}(\mathrm{t}) + \left(\frac{\mathrm{EJ}}{\mu}\int_0^L \mathrm{U_m^{IV}}(\mathrm{x})\mathrm{U_k}(\mathrm{x})\mathrm{dx}\right)\mathrm{Y_m}(\mathrm{t}) - \left(\frac{\mathrm{N}}{\mu}\int_0^L \mathrm{U_m^{II}}(\mathrm{x})\mathrm{U_k}(\mathrm{x})\mathrm{dx}\right)\mathrm{Y_m}(\mathrm{t})\right.$$
$$+\left(\frac{\mathrm{K_0}}{\mu}\int_0^L \mathrm{U_m}(\mathrm{x})\mathrm{U_k}(\mathrm{x})\mathrm{dx}\right)\mathrm{Y_m}(\mathrm{t}) - \left(\frac{\mathrm{G_0}}{\mu}\int_0^L \mathrm{U_m^{II}}(\mathrm{x})\mathrm{U_K}(\mathrm{x})\right)\mathrm{Y_m}(\mathrm{t})$$
$$+\frac{\mathrm{M_i}}{\mu}\left[\left(\int_0^L \cos\omega\mathrm{t}\mathrm{H}(\mathrm{x}-\mathrm{ct})\mathrm{U_m}(\mathrm{x})\mathrm{U_k}(\mathrm{t})\mathrm{dx}\right)\ddot{\mathrm{Y}}_\mathrm{m}(\mathrm{t})\right.$$
$$+2\mathrm{c}\left(\int_0^L \cos\omega\mathrm{t}\mathrm{H}(\mathrm{x}-\mathrm{ct})\mathrm{U_m}(\mathrm{x})\mathrm{U_k}(\mathrm{x})\mathrm{dx}\right)\dot{\mathrm{Y}}_\mathrm{m}(\mathrm{t})$$
$$\left.\left.+\mathrm{c}^2\left(\int_0^L \cos\omega\mathrm{t}\mathrm{H}(\mathrm{x}-\mathrm{ct})\,\mathrm{U_m}(\mathrm{x})\mathrm{U_k}(\mathrm{x})\mathrm{dx}\right)\mathrm{Y_m}(\mathrm{t})\mathrm{dx}\right]\right\}$$
$$=\frac{\mathrm{M_i}}{\mu}\mathrm{g}\int_0^L \cos\omega\mathrm{t}\mathrm{H}(\mathrm{x}-\mathrm{ct})\mathrm{U_k}(\mathrm{x})\mathrm{dx} \tag{11}$$

Equation (11) can be re-written as

$$\sum_{m=1}^{n}\left\{A_1(m,k)\ddot{Y}_m(t) + [A_2(m,k) - A_3(m,k) + A_4(m,k) - A_5(m,k)]Y_m(t)\right.$$
$$\left.+\frac{M_i g}{\mu L}[LA_6(m,k)\ddot{Y}_m(t) + 2cLA_7(m,k)\dot{Y}_m(t) + c^2LA_8(m,k)Y_m(t)]\right\}$$
$$=\frac{M_i g}{\mu}\int_0^L \cos\omega t H(x - ct)\, u_k dx \tag{12}$$

$$A_1(m,k) = \int_0^L \mathrm{U_m}(x)\mathrm{U_k}(x)\mathrm{d}x, \quad A_2(m,k) = \frac{EJ}{\mu}\int_0^L \mathrm{U_m^{IV}}(x)\mathrm{U_k}(x)\mathrm{d}x \tag{13}$$

$$A_3(m,k) = \frac{N}{\mu}\int_0^L \mathrm{U_m^{II}}(x)\mathrm{U_k}(x)\mathrm{d}x, A_4(m,k) = \frac{K_0}{\mu}\int_0^L \mathrm{U_m}(x)\mathrm{U_k}(x)\mathrm{d}x \tag{14}$$

$$A_5(m,k) = \frac{G_0}{\mu}\int_0^L \mathrm{U_m^{II}}(x)\mathrm{U_k}(x)\mathrm{d}x \;\; , A_6(m,k) = \int_0^L \cos\omega t H(\,x - ct)U_m(x)U_k(x)dx \tag{15}$$

$$A_7(m,k) = \int_0^L \cos\omega t H(\,x - ct)U_m^I(x)U_k(x)dx \tag{16}$$

$$A_8(m,k) = \int_0^L \cos\omega t H(x-ct) U_m^{II}(x) U_k(x) dx \tag{17}$$

$$A_9(m,k) = \int_0^L \cos\omega t H(x-ct) U_k(x) dx \tag{18}$$

In order to evaluate the integrals $A_6(m,k), A_7(m,k), A_8(m,k),$ and $A_9(m,k)$, one makes use of the Fourier series representation for the Heaviside function in the form;

$$H(x-ct) = \frac{1}{4} + \frac{1}{\pi}\sum_{n=0}^{\infty} \frac{\sin\big((2n+1)\pi(x-ct)\big)}{2n+1}, \qquad 0 < x < 1 \tag{19}$$

Substituting (19) in (10), after some simplifications and rearrangements, one obtains

$$\begin{aligned}
\sum_{m=1}^{n} \{ & A_1(m,k)\ddot{Y}_m(t) + Q_0(m,k)Y_m(t) + \lambda_0 \cos\omega t[(\frac{1}{4}\Delta_1(m,k) \\
& + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_2(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_3(n,m,k))\ddot{Y}_m(t) \\
& + 2c\Bigg(\frac{1}{4}\Delta_4(m,k) \\
& + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_5(n,m,k) \\
& - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_6(n,m,k)\Bigg)\dot{Y}_m(t) + c^2\Bigg(\frac{1}{4}\Delta_7(m,k) \\
& + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi ct}{2n+1}\Delta_8(n,m,k) - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi ct}{2n+1}\Delta_9(n,m,k)\Bigg)Y_m(t)]
\end{aligned}$$

$$\begin{aligned}
= \frac{\mathrm{M}gL\cos\omega t}{\mu\lambda_k}\Big[& -\cos\lambda_k + A_k\sin\lambda_k + B_k\cosh\lambda_k + C_k\sinh\lambda_k + \cos\frac{\lambda_k ct}{L} \\
& - A_k\sin\frac{\lambda_k ct}{L} - B_k\cosh\frac{\lambda_k ct}{L} - C_k\sinh\frac{\lambda_k ct}{L}\Big]
\end{aligned} \tag{20}$$

where,

$$Q_0(m,k) = A_2(m,k) - A_3(m,k) + A_4(m,k) - A_5(m,k) \tag{21}$$

$$\lambda_0 = \frac{\mathrm{M}}{\mu \mathrm{L}} \tag{22}$$

Equation (20) is the transformed equation governing the problem of the dynamic response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on bi-parametric elastic foundation. This coupled non-homogeneous second order ordinary differential equation holds for all variant of the classical boundary conditions.

3.1 Case I: Moving force problem

In moving force, we account for only the load being transferred to the structure. In this case, the inertia effect is negligible. Setting $\lambda_0 = 0$ in the transformed equation (20), one obtains

$$\sum_{m=1}^{n}\{A_1(m,k)\ddot{Y}_m(t) + Q_0(m,k)Y_m(t)\}$$
$$= \frac{M_i gL\cos\omega t}{\mu\lambda_k}\Big[-\cos\lambda_k + A_k\sin\lambda_k + B_k\cosh\lambda_k + \lambda_k c_k\sinh + \cos\frac{\lambda_k ct}{L} - A_k\sin\frac{\lambda_k ct}{L} - B_k\cosh\frac{\lambda_k ct}{L} - C_k\sinh\frac{\lambda_k ct}{L}\Big] \quad (23)$$

Equation (23)is an approximate model of the differential equation describing the response of Bernoulli-Euler beam with general boundary conditions when under the action of moving distributed force which assumes the inertia effect of the moving mass as negligible.

Further rearrangement of (23) yields

$$\ddot{Y}_m(t) + \beta_p^2 Y_m(t) = F_m\cos\omega t\,[-\cos\lambda_k + A_k\sin\lambda_k + B_k\cosh\lambda_k + C_k\sinh\lambda_k + \cos\alpha_y t - A_k\sin\alpha_y t - B_k\cosh\alpha_y t - C_k\sinh\alpha_y t] \quad (24)$$

where,

$$F_m = \frac{M_i gL}{\mu\lambda_k A_1(m,k)} \quad (25)$$

$$\beta_p^2 = \frac{Q_0(m,k)}{A_1(m,k)} \quad (26)$$

$$\alpha_{y=} \frac{\lambda_k c}{L} \quad (27)$$

Solving equation (24) using Laplace transformation and convolution theory and taking into account equation (8), one obtains.

$$W(x,t) = \sum_{m=1}^{n}\frac{\mathrm{MgL}}{\mu\lambda_k A_1(m,k)} \times \frac{\cos\omega t}{\beta_p^2(\beta_p^4 - \alpha_y^4)}\{\theta_{mc}(1-\cos\beta_p t)(\beta_p^4 - \alpha_y^4) + (\beta_p^2 - \alpha_y^2)\left[\beta_p^2(\cos\beta_p t - \cos\alpha_y t) - A_k\beta_p(\alpha_y\sin\beta_p t - \beta_y\sin\alpha_y t)\right] - (\beta_p^2 - \alpha_y^2)\left[B_k\beta_p^2(\cosh\alpha_y t - \cos\beta_p t) + C_k\beta_p(\beta_p\sinh\alpha_y t - \alpha_y\sin\beta_p t)\right]\} \times [\sin\frac{\lambda_k x}{L} + A_k\cos\frac{\lambda_k x}{L} + B_k\sinh\frac{\lambda_k x}{L} + C_k\cosh\frac{\lambda_k x}{L}] \quad (28)$$

Equation (28) represents the transverse-displacement response to a variable-magnitude moving force of a Bernoulli-Euler beam resting on a bi-parametric elastic foundation.

3.2 Case II: Moving mass problem

If the mass of the structure and that of the load are of comparable magnitude, the inertia effect of the moving mass is not negligible. Thus, $\lambda_0 \neq 0$ and solution to the entire equation (20) is required. This is termed moving mass problem. To this end, equation (20) is rearranged to take the form

$$\ddot{Y}_m(t) + \frac{2c\lambda_0 Z_2(r,m,v)\cos\omega t}{A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t}\dot{Y}_m(t) + \frac{Q_0(m,k) + \lambda_0 c^2 Z_3(r,m,v)\cos\omega t}{A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t}Y_m(t)$$
$$= \frac{\lambda_0 gL\cos\omega t}{\lambda_k(A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t)}[-\cos\lambda_k + A_k\sin\lambda_k + B_k\cosh\lambda_k + C_k\sinh\lambda_k + \cos\frac{\lambda_k ct}{L} - A_k\sin\frac{\lambda_k ct}{L} - B_k\cosh\frac{\lambda_k ct}{L} - C_k\sinh\frac{\lambda_k ct}{L}] \quad (29)$$

Where,

$$Z_1(r,m,v) = \frac{\Delta_1(m,k)}{4} + \frac{1}{\pi}\sum_n^\infty \frac{\cos(2n+1)\pi ct}{2n+1}\Delta_2(n,m,k) - \frac{1}{\pi}\sum_n^\infty \frac{\sin(2n+1)\pi ct}{2n+1}\Delta_3(n,m,k) \quad (30)$$

$$Z_2(r,m,v) = \frac{\Delta_4(m,k)}{4} + \frac{1}{\pi}\sum_n^\infty \frac{\cos(2n+1)\pi ct}{2n+1}\Delta_5(n,m,k) - \frac{1}{\pi}\sum_n^\infty \frac{\sin(2n+1)\pi ct}{2n+1}\Delta_6(n,m,k) \quad (31)$$

$$Z_3(r,m,v) = \frac{\Delta_7(m,k)}{4} + \frac{1}{\pi}\sum_n^\infty \frac{\cos(2n+1)\pi ct}{2n+1}\Delta_8(n,m,k) - \frac{1}{\pi}\sum_n^\infty \frac{\sin(2n+1)\pi ct}{2n+1}\Delta_9(n,m,k) \quad (32)$$

Evidently, unlike the moving force problem, an exact analytical solution to the equation (29) does not exist, and there is no known approximate analytical solution technique that can be used to solve the equation, even the popular struble's technique cannot simplify the equation due to the variability of the load magnitude. Hence, one resorts to numerical technique and to this end use is made of Runge-Kutta of fourth order to solve the second order coupled ordinary differential equation. We now proceed to use Runge-Kutta of fourth order.

The second order ordinary differential equation (29) is first reduced to two systems of first order as follow:

$$\dot{Y}_m(t) = Z_m \quad (33)$$

$$\dot{Z}_m(t) = DC_4 - DC_2 Z_m - DC_3 Y_m(t) \quad (34)$$

Where,

$$DC_2 = \frac{2c\lambda_0 Z_2(r,m,v)\cos\omega t}{A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t} \quad (35)$$

$$DC_3 = \frac{Q_0(m,k) + \lambda_0 c^2 Z_3(r,m,v)\cos\omega t}{A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t} \quad (36)$$

$$DC_4 = \frac{\lambda_0 gL\cos\omega t}{\lambda_k(A_1(m,k) + \lambda_0 Z_1(r,m,v)\cos\omega t)}[-\cos\lambda_k + A_k\sin\lambda_k + B_k\cosh\lambda_k + C_k\sinh\lambda_k + \cos\frac{\lambda_k ct}{L} - A_k\sin\frac{\lambda_k ct}{L} - B_k\cosh\frac{\lambda_k ct}{L} - C_k\sinh\frac{\lambda_k ct}{L}] \quad (37)$$

The fourth order Runge-Kutta scheme is given by

$$y_{m+1} = y_m + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (38)$$

$$z_{m+1} = z_m + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \quad (39)$$

Where,

$$k_1 = hf(x_m, y_m, z_m)$$

$$l_1 = hg(x_m, y_m, z_m)$$

$$k_2 = hf\left(x_m + \frac{h}{2}, y_m + \frac{k_1}{2}, z_m + \frac{l_1}{2}\right)$$

$$l_2 = hg\left(x_m + \frac{h}{2}, y_m + \frac{k_1}{2}, z_m + \frac{l_1}{2}\right)$$

$$k_3 = hf\left(x_m + \frac{h}{2}, y_m + \frac{k_2}{2}, z_m + \frac{l_2}{2}\right)$$

$$l_3 = hg\left(x_m + \frac{h}{2}, y_m + \frac{k_2}{2}, z_m + \frac{l_2}{2}\right)$$

$$k_4 = hf(x_m + h, y_m + k_3, z_m + l_3)$$

$$l_4 = hg(x_m + h, y_m + k_3, z_m + l_3) \quad (40)$$

4 Discussion of the Analytical Solution and Numerical Solution

For thisundamped system, it is desirable to examine the phenomenon of resonance. From equation (28), it is clearly shown that the uniform Bernoulli-Euler beam resting on bi-parametric elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance whenever

$$\beta_p = \alpha_y \quad (41)$$

where

$$\alpha_y = \frac{\lambda_k c}{L} \quad (42)$$

That is,

$$\beta_p = \frac{\lambda_k c}{L} \quad (43)$$

For the solution of the moving distributed mass problem, the problem is not solvable by any conventional method, even the popular struble's technique could not simplify the transformed governing coupled differential equation (29), and hence the fourth order Runge-Kutta scheme is used to obtain the numerical solution of the moving distributed mass problem. The Runge-Kutta scheme of order four is used to solve the moving distributed force problem and the results are shown to compare favourably with the analytical results of the moving force problem thereby confirming the accuracy of the Runge-Kutta scheme in solving this kind of dynamical problem.

5 Illustrative Examples

5.1 Simply supported boundary conditions

As an example, we consider a uniform beam simply support at the ends $x = 0$ and elastically supported at the other end $x = L$, the conditions are expressed as

$$W(0,t) = W(L,t) \quad , \qquad \frac{\partial^2 W(0,t)}{\partial x^2} = 0 = \frac{\partial^2 W(L,t)}{\partial x^2} \tag{44}$$

and hence for the normal modes

$$U_m(0) = 0 = U_m(L) \;\; , \;\; \frac{\partial^2 U_m(0)}{\partial x^2} = 0 = \frac{\partial^2 U_m(L)}{\partial x^2} \tag{45}$$

this implies that

$$U_k(0) = 0 = U_k(L) \; , \;\; \frac{\partial^2 U_k(0)}{\partial x^2} = 0 = \frac{\partial^2 U_k(L)}{\partial x^2} \tag{46}$$

Thus, it can be shown that

$$A_m = A_k = 0 \; ; \;\; A_m = A_k = 0 \; ; \;\; C_m = C_k = 0 \tag{47}$$

$$\lambda_k = k\pi \text{ and } \quad \lambda_m = m\pi \tag{48}$$

5.2 Clamped end boundary conditions

At a clamped end, both deflection and slope vanish. Thus,

$$W(0,t) = 0 = W(L,t) \text{and } \frac{\partial W(0,t)}{\partial x} = 0 = \frac{\partial W(L,t)}{\partial x} \tag{49}$$

Hence for normal modes

$$U_m(0,t) = 0 = U_m(L) \; and \;\; \frac{\partial U_m(0,t)}{\partial x} = 0 = \frac{\partial U_m(L)}{\partial x} \tag{50}$$

Which implies that

$$U_k(0,t) = 0 = U_k(L) \;\; and \quad \frac{\partial U_k(0,t)}{\partial x} = 0 = \frac{\partial U_k(L)}{\partial x} \tag{51}$$

Thus it can be shown that

$$A_m = \frac{\sinh\lambda_m - \sin\lambda_m}{\cos\lambda_m - \cosh\lambda_m} = \frac{\cos\lambda_m - \cosh\lambda_m}{\sinh\lambda_m + \sin\lambda_m} = -C_m \qquad and \quad B_m = -1 \tag{52}$$

The frequency equation is given as

$$\cos\lambda_m \cosh\lambda_m = 1 \tag{53}$$

Hence, we have

$$\lambda_1 = 4.73004,\ \ \lambda_2 = 7.85320, \qquad \lambda_3 = 10.99561 \tag{54}$$

Expression for $A_{k,}B_{k,}\ \ C_{k,}$ and the corresponding frequency equation are obtained by a simple interchange of m and k in (52)

5.3 Free end boundary conditions

For free ends condition at x=0 and x=L, the pertinent boundary conditions are

$$\frac{\partial^2 U(0,t)}{\partial x^2} = 0 = \frac{\partial^2 U(L,t)}{\partial x^2}\ and\ \frac{\partial^3 U(0,t)}{\partial x^3} = 0\frac{\partial^3 U(L,t)}{\partial x^3} \tag{55}$$

Thus, for normal modes

$$\frac{d^2 U_m(0)}{dx^2} = 0 = \frac{d^2 U_m(L)}{dx^2}\ and\ \frac{d^2 U_m(0)}{dx^2} = 0 = \frac{d^3 U_m(L)}{dx^3} \tag{56}$$

Which implies that

$$\frac{d^2 U_k(0)}{dx^2} = 0 = \frac{d^2 U_k(L)}{dx^2}\ and\ \frac{d^2 U_k(0)}{dx^2} = 0 = \frac{d^3 U_k(L)}{dx^3} \tag{57}$$

Thus, it can be shown that

$$A_m = \frac{\sin\lambda_m - \sinh\lambda_m}{\cosh\lambda_m - \cos\lambda_m} = \frac{\cos\lambda_m - \cosh\lambda_m}{\sin\lambda_m + \sinh\lambda_m} = C_m\ and\ B_m = 1 \tag{58}$$

and from (4.24), one obtains

$$\cos\lambda_m \cosh^{\lambda_m} = 1 \tag{59}$$

Which is termed the frequency equation for the dynamical problem, such that

$$\lambda_1 = 4.73004,\ \ \lambda_2 = 7.85320, \qquad \lambda_3 = 10.99561 \tag{60}$$

6 Numerical Results and Discussion

6.1 Graphs for simply supported boundary conditions

Figs. 6.1 and 6.2 display the effect of axial force **N** on the deflection profile of simply supported uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the axial force increases.

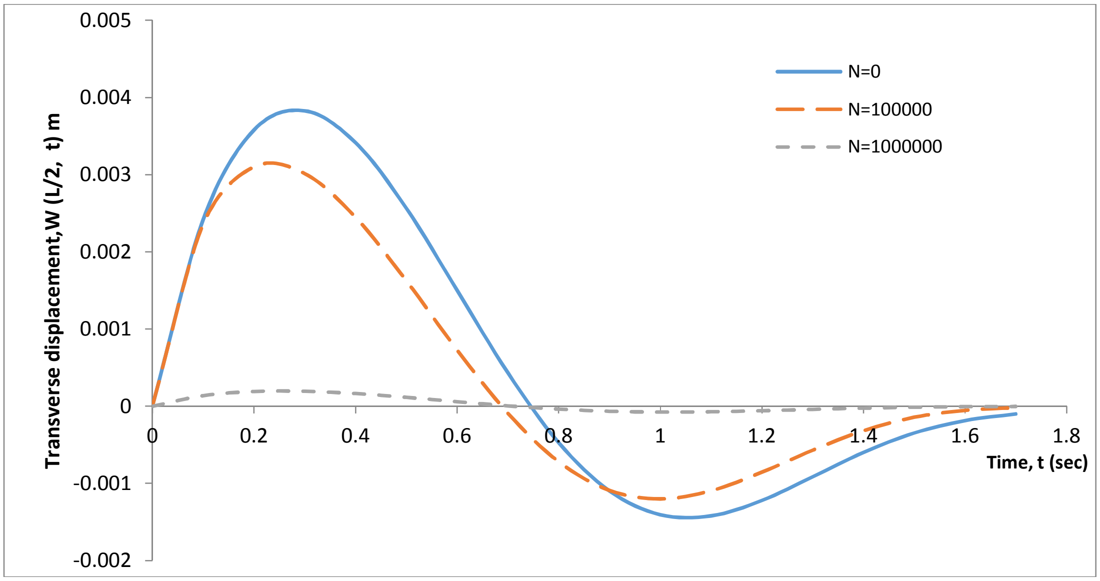

Fig. 6.1. Deflection profile of a simply supported uniform beam under moving distributed force for various values of axial force N for fixed values of shear modulus G_0= 100000 and foundation modulus K_0 = 400000

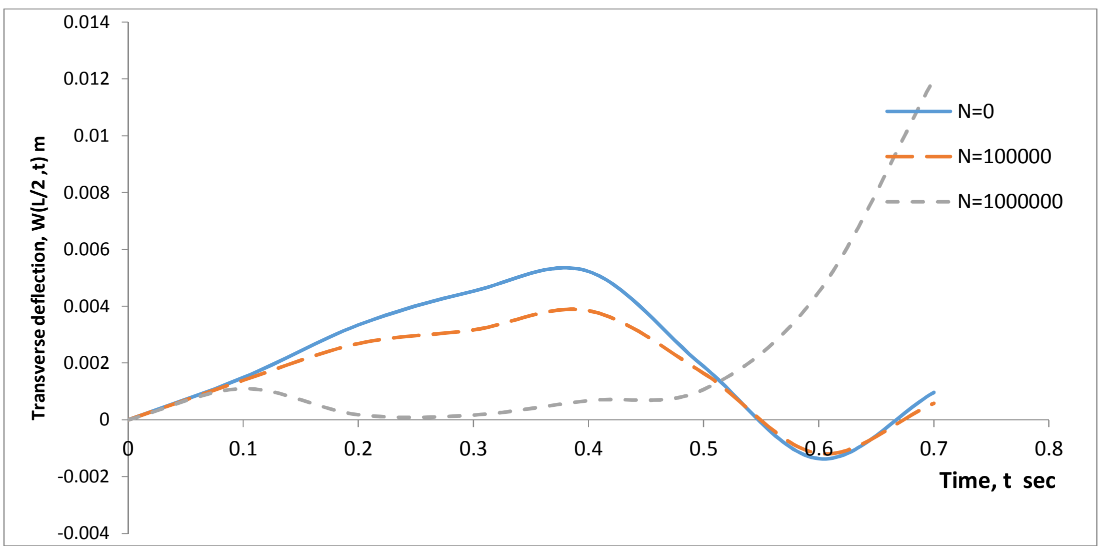

Fig. 6.2. Deflection profile of a simply supported uniform beam under moving distributed mass for various values of axial force N for fixed values of shear modulus G_0= 100000 and foundation modulus K_0 = 400000

Figs. 6.3 and 6.4 display the effect of shear modulus $\mathbf{G_0}$ on the deflection profile of simply supported uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the shear modulus $\mathbf{G_0}$ increases.

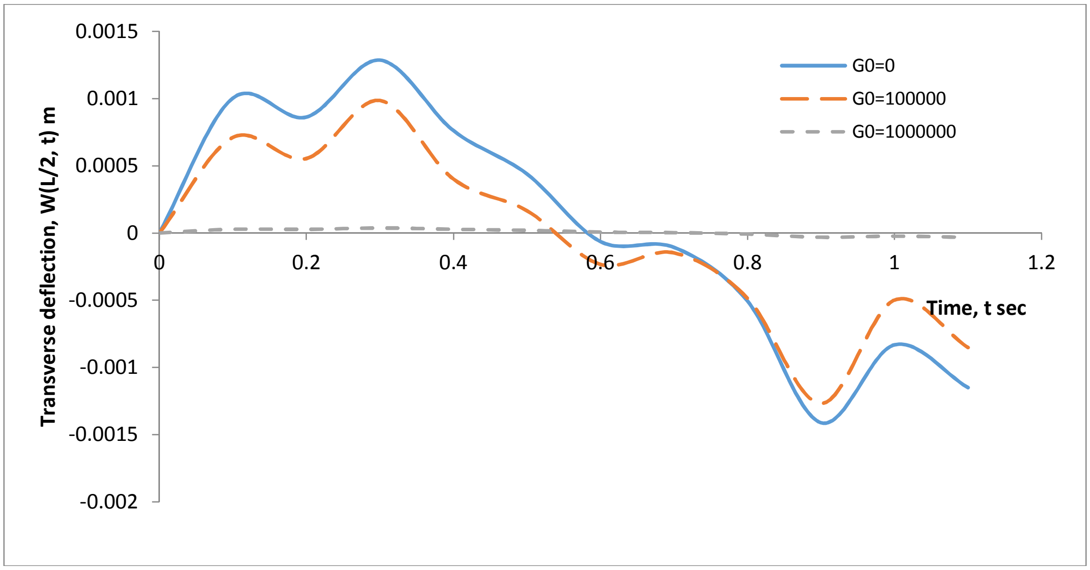

Fig. 6.3. Deflection profile of a simply supported uniform beam under moving distributed force for various values of shear modulus G_0 for fixed values of shear modulus N = 24000 and foundation modulus K_0 = 400000.

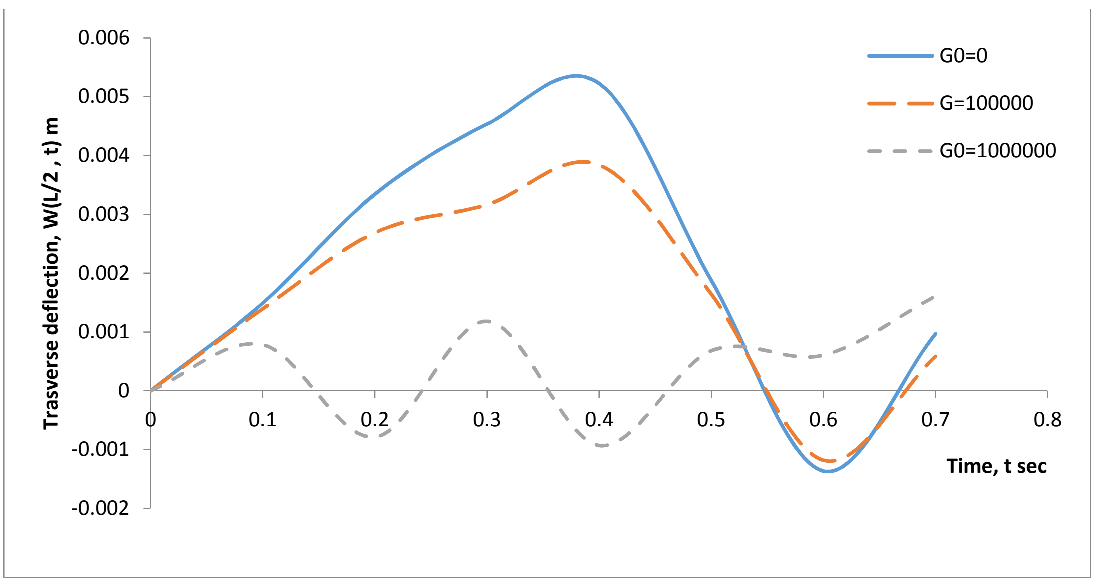

Fig. 6.4. Deflection profile of a simply supported uniform beam under moving distributed mass for various values of shear modulus G_0 for fixed values of axial force N= 24000 and foundation modulus K_0 = 400000

Figs. 6.5 and 6.6 display the effect of foundation modulus $\mathbf{K_0}$ on the deflection profile of simply supported uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the foundation modulus $\mathbf{K_0}$ increases.

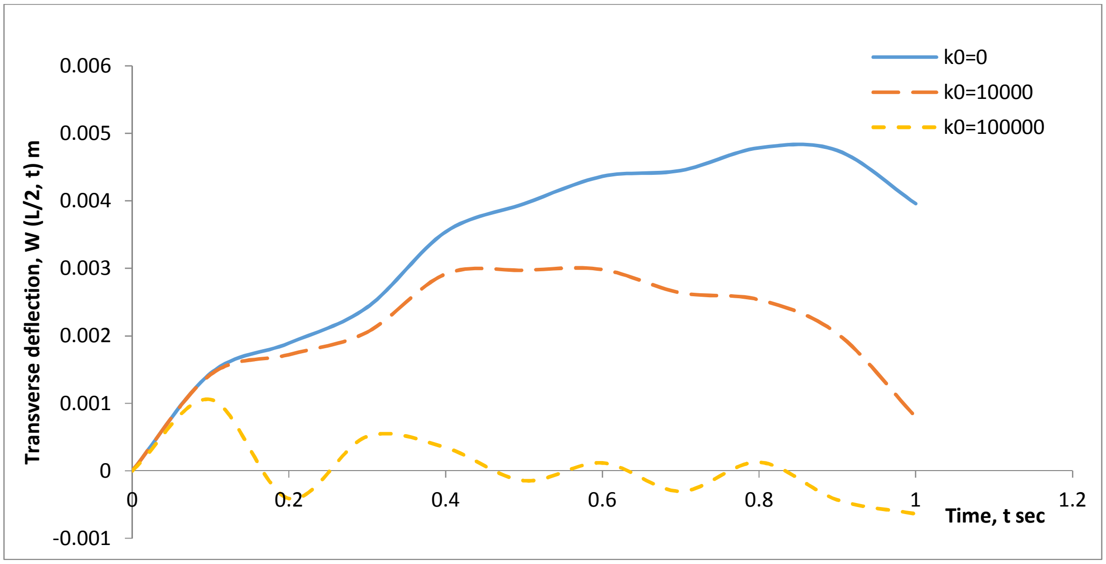

Fig. 6.5. Deflection profile of a simply supported uniform beam under moving distributed force for various values of foundation modulus K_0 for fixed values of axial force N= 10000 and foundation modulus G_0= 100000

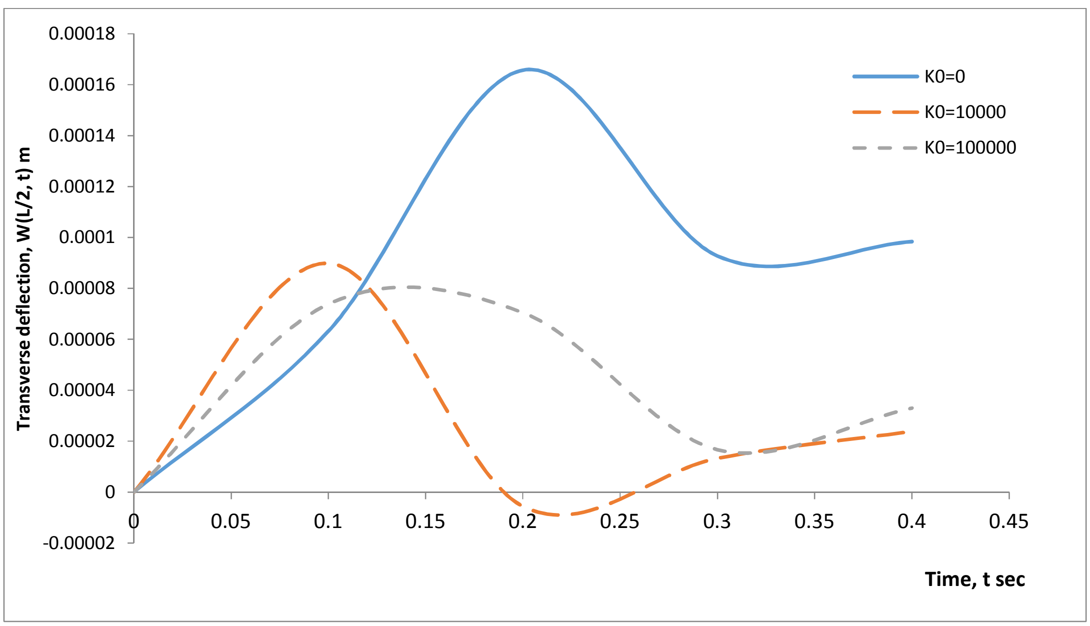

Fig. 6.6. Deflection profile of a simply supported uniform beam under moving distributed mass for various values of Foundation modulus $K_0$for fixed values of axial force N= 10000 and Foundation modulus G_0 = 10000

Fig. 6.7 shows thecomparison of the moving distributed forces and moving distributed masses for fixed the values of N, K_0 and G_0.

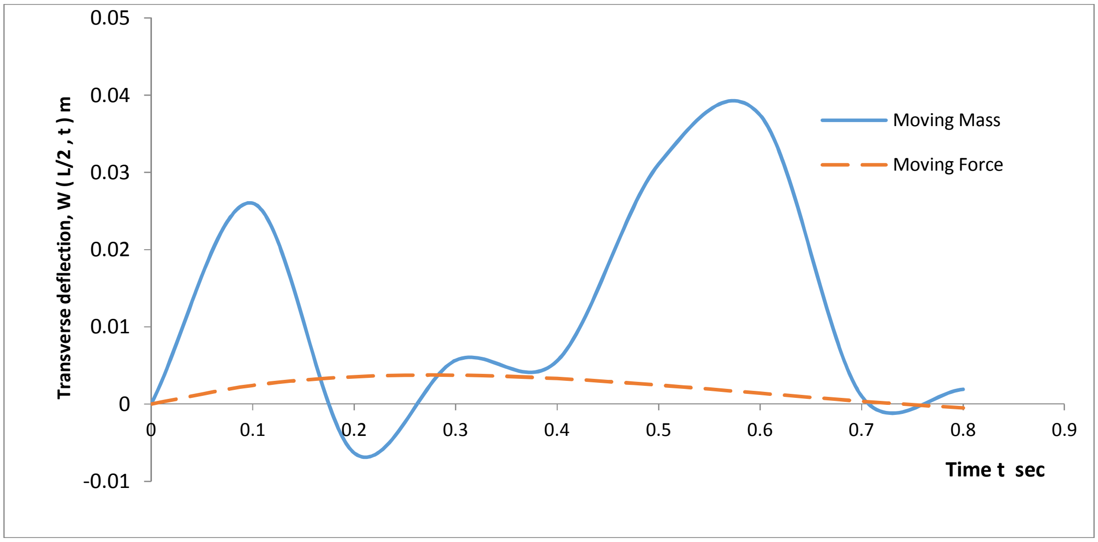

Fig. 6.7. Comparison of the deflection profile of moving force and moving mass for a simply supported uniform beam for fixed values of N, K_0 and G_0.

6.2 Graphs for clamped end boundary conditions

Figs. 6.8 and 6.9 display the effect of foundation modulus $\mathbf{K_0}$ on the deflection profile of clamped end uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the foundation modulus $\mathbf{K_0}$ increases.

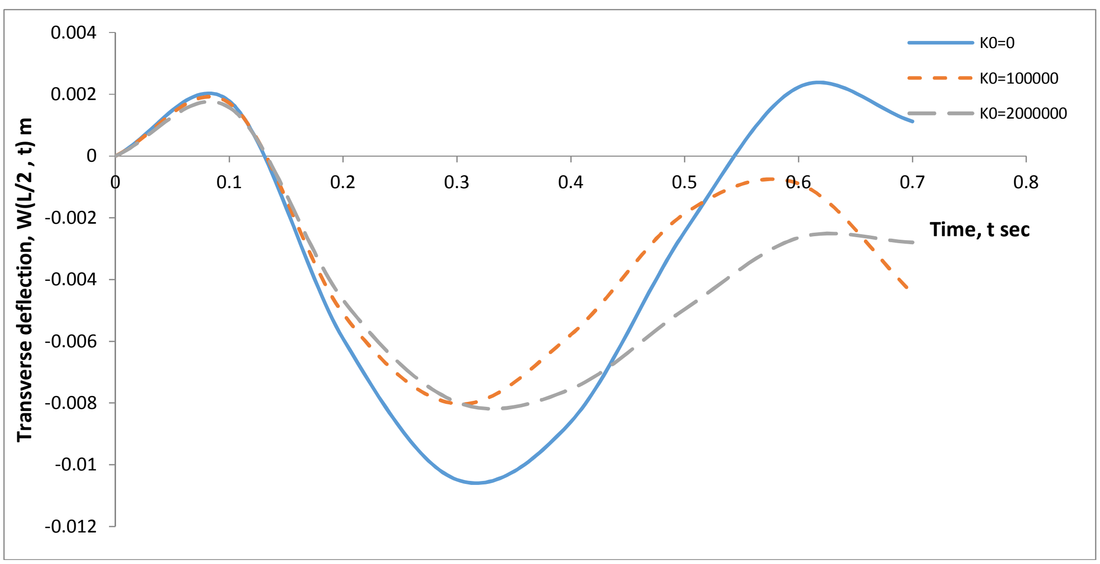

Fig. 6.8. Deflection profile of a clamped end uniform beam under moving distributed force for various values of foundation modulus $K_0$for fixed values of axial force N= 100000 and shear modulus G_0 = 1000000

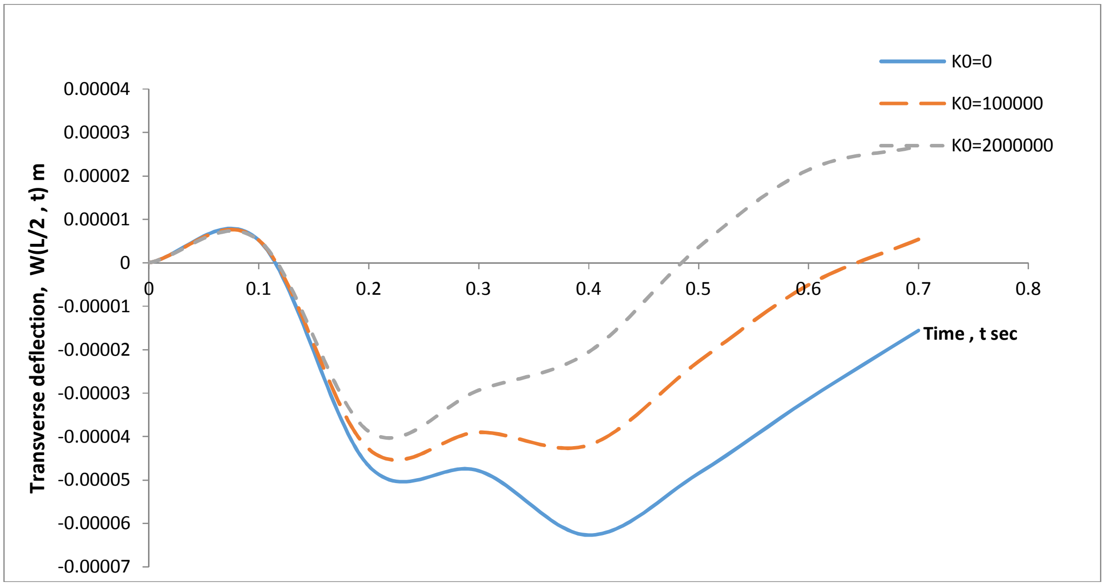

Fig. 6.9. Deflection profile of a clamped end uniform beam under moving distributed massfor various values of foundation modulus $K_0$for fixed values of axial force N= 100000 and shear modulus G_0 = 1000000

Figs. 6.10 and 6.11 display the effect of shear modulus G_0 on the deflection profile of clamped end uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass. The graphs show that the response amplitude decreases as the value of the shear modulus G_0 increases.

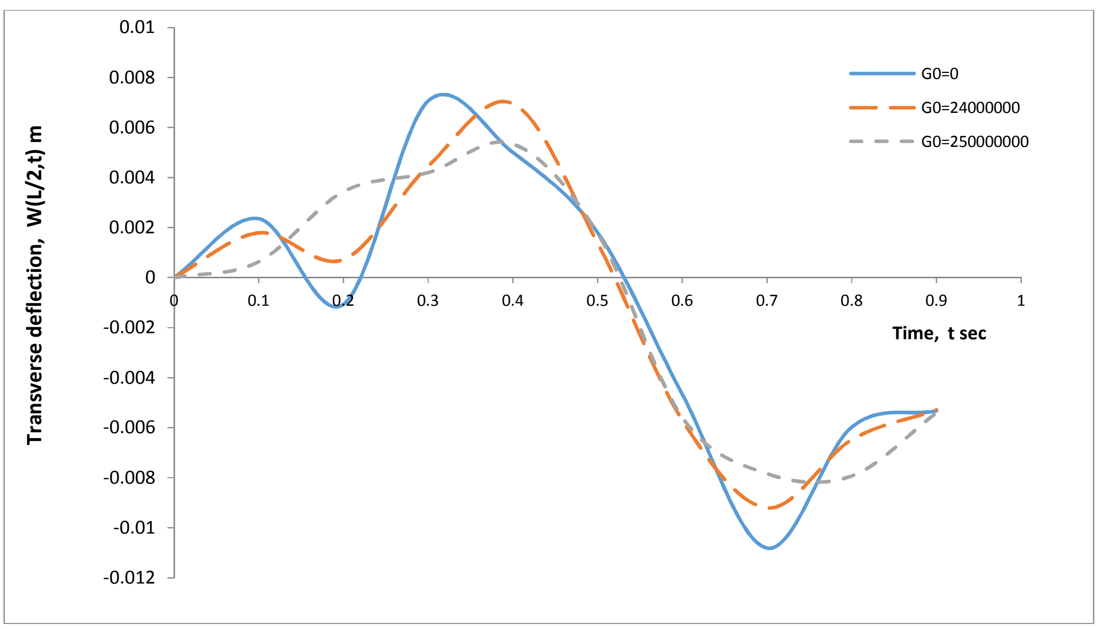

Fig. 6.10. Deflection profile of a clamped end uniform beam under moving force for various values of shear modulus $G_0$for fixed values of axial force N= 100000 and foundation modulus K_0 = 1000000

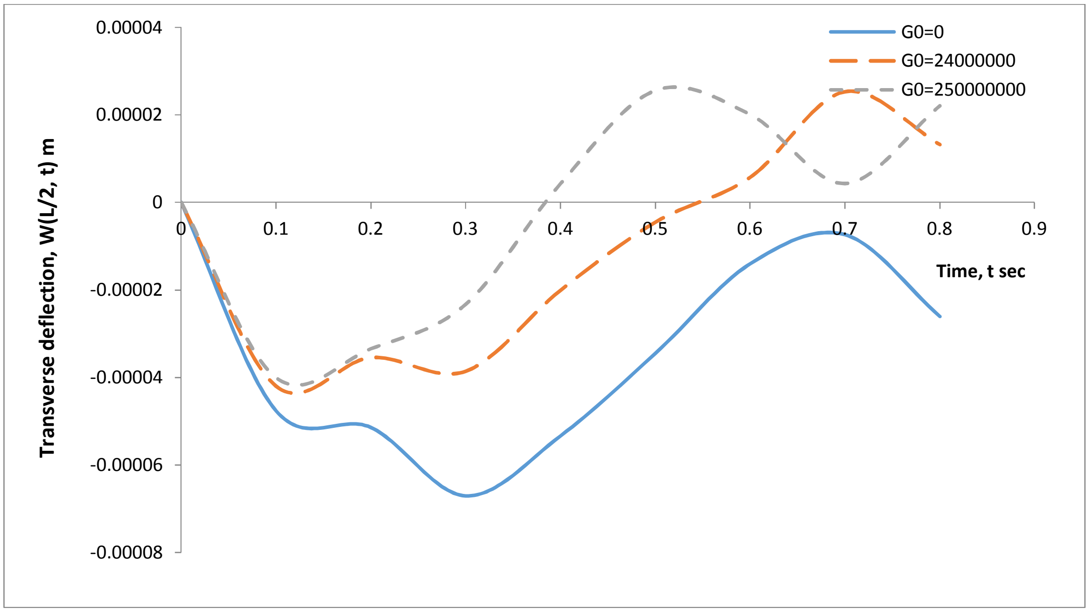

Fig. 6.11. Deflection profile of a clamped end uniform beam under moving mass for various values of shear modulus $G_0$for fixed values of axial force N= 100000 and foundation modulus K_0 = 1000000

Figs. 6.12 and 6.13 display the effect of axial force **N** on the deflection profile of clamped end uniform beam under the action of variable-magnitude distributed forces moving at constant velocity in both cases of moving distributed force and moving distributed mass respectively. The graphs show that the response amplitude decreases as the value of the axial **N** force increases.

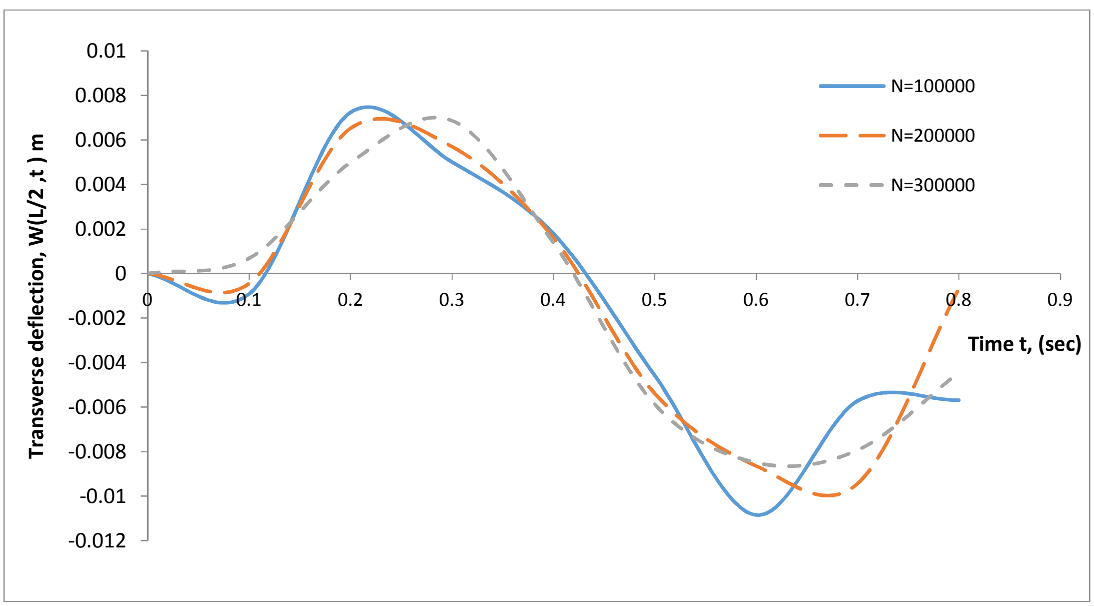

Fig. 6.12. Deflection profile of a clamped end uniform beam under moving distributed force for various values of axial force N for fixed values of foundation modulusK_0= 100000 and shear modulus G_0 = 100000

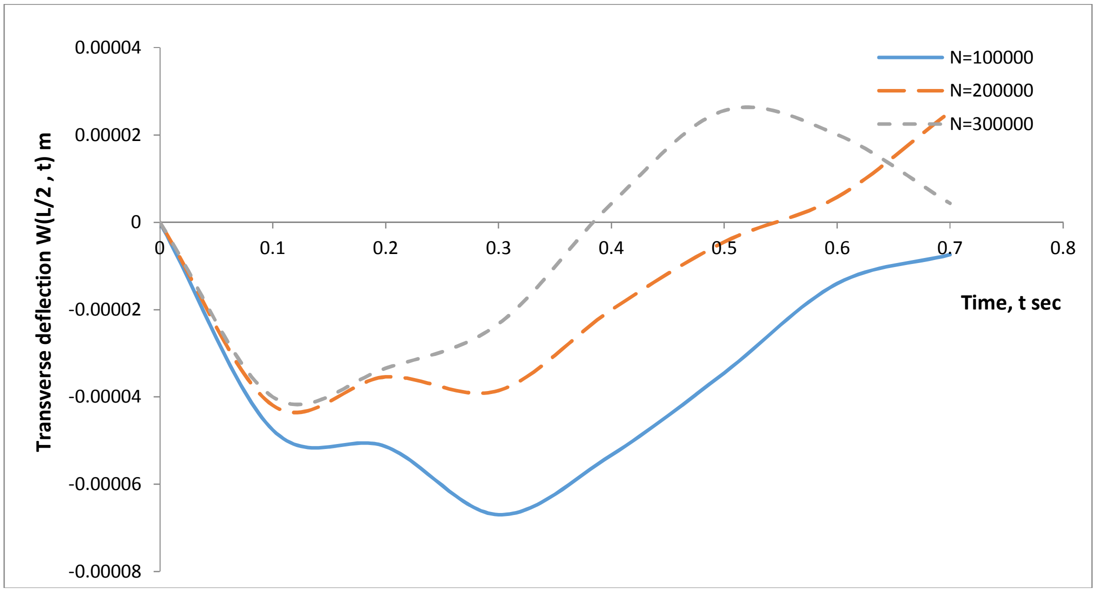

Fig. 6.13. Deflection profile of a clamped end uniform beam under moving distributed mass for various values of axial force N for fixed values of foundation modulusK_0= 100000 and shear modulus G_0 = 100000

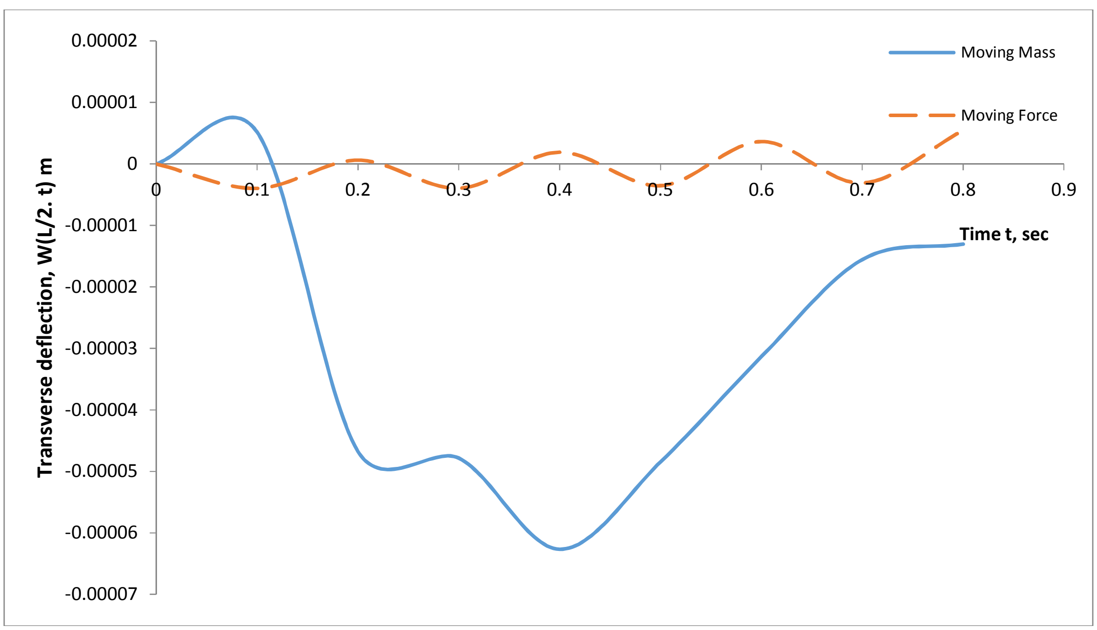

Fig. 6.14. Shows the comparison of the moving distributed forces and moving distributed masses for fixed values of N, K_0 and G_0

Fig. 6.15 shows the comparison of the displacement response of the moving distributed force for analytical solution and numerical solution for a free ends uniform beam.

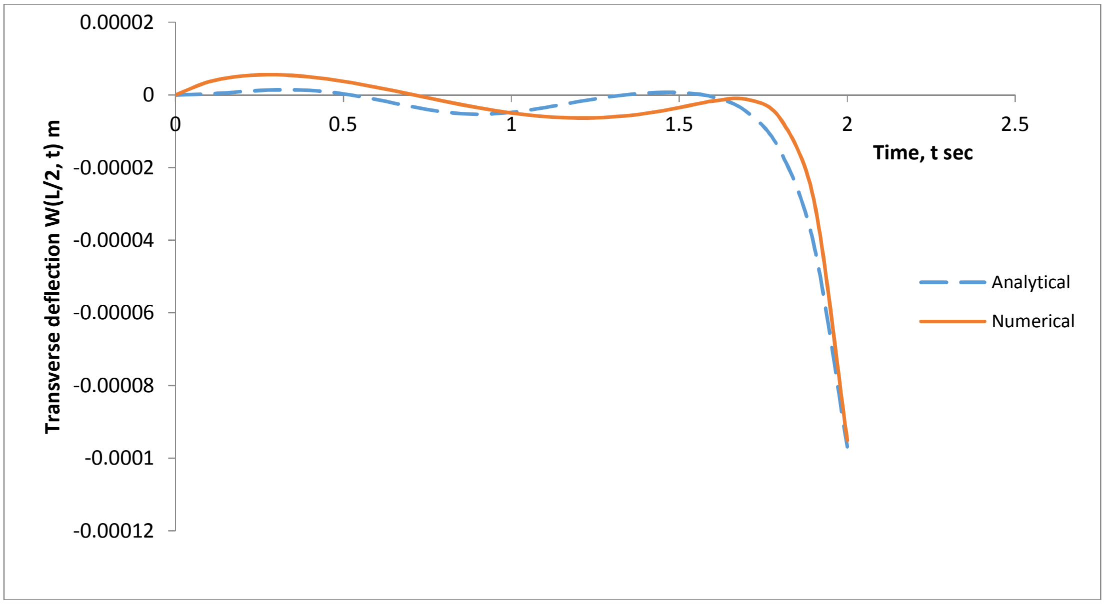

Fig. 6.15. Comparison of the displacement response of the moving distributed force for analytical solution and numerical solution for a free ends uniform beam

7 Conclusions

The problem of assessing the dynamic response to variable-magnitude moving distributed masses of Bernoulli-Euler beam resting on bi-parametric elastic foundation is considered. The close form solution of the governing fourth order partial differential equation with variable and singular coefficients of uniform Bernoulli-Euler beam for moving force is presented. Firstly, the Galerkin's method is used to transform the governing fourth order partial differential equation with singular and variable coefficients to a set of coupled second order ordinary differential equations called the Galerkin's equations. The resulting Galerkin's equations are solved, for the moving force problem, using Laplace transform and convolution theory.For the solution of the moving mass problem, the problem is not solvable by any conventional method, even the popular struble's technique could not simplify the transformed governing coupled differential equation, and hence the fourth order Runge-Kutta scheme is used to obtain the numerical solution of the moving mass problem. The Runge-Kutta scheme of order four is used to solve the moving force problem and the results are shown to compare favourably with the analytical results of the moving force problem thereby confirming the accuracy of the Runge-Kutta scheme in solving this kind of dynamical problem.The results show that response amplitude of the Bernoulli-Euler beam under variable-magnitude moving load decrease as the axial force N increases for all variants of classical boundary conditions considered. For fixed value of N, the displacements of the beam resting on bi-parametric elastic foundation decrease as the foundation modulus K_0 increases. Furthermore, as the shear modulus G_0 increases, the transverse deflections of the beam decrease.

Competing Interests

Authors have declared that no competing interests exist.

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