Asian Research Journal of Mathematics

5(1): 1-11, 2017; Article no.ARJOM.31585

ISSN: 2456-477X

MHD Natural Convective Oscillatory Flow in Bifurcating Blood Capillaries

W. I. A. Okuyade[1*]

[1]Department of Mathematics and Statistics, University of Port Harcourt, Port Harcourt, Nigeria.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/31585

Editor(s):

(1) Xingting Wang, Department of Mathematics, Temple University, Philadelphia, USA.

Reviewers:

(1) Moses O. Oyesanya, University of Nigeria, Nsukka, Nigeria.

(2) Mohamed Deghmoum, University of M'HAMMED BOUGARA Boumerdes, Algeria.

(3) Mohammad Mainul Hoque Pavel, University of Newcastle, Australia.

(4) P. Sulochana, Intell Engineering College, Anantapuramu, India.

Complete Peer review History: http://www.sciencedomain.org/review-history/19866

Original Research Article

Received: 15th January 2017
Accepted: 2nd March 2017
Published: 4th July 2017

Abstract

Oscillatory flow in bifurcating blood capillaries is presented. The governing nonlinear and coupled equations expressed in the form of the Boussinesq approximations are solved by the method of perturbation series expansions. Solutions for the concentration, temperature and velocity are obtained, and presented quantitatively using Malple 18 computational soft ware. The results show that the rate of chemical reaction, Hartmann number ($M^2 \leq I.0$), heat exchange parameter and Grashof number ($Gr/Gc \leq I.0$) tend to increase the velocity of the flow. The increase in the velocity structure has some attendant implications. In fact, it tends to increase the rate of transport of oxygen and nutrient-rich blood to the tissues, and this in turn enhances the physiological well-being of man.

Keywords: Blood flow; bifurcation; capillaries; oscillatory flow.

**Corresponding author: E-mail: wilsonia6011@gmail.com, wiaokuyade@gmail.com;*

1 Introduction

The study of blood flow in the capillaries has received considerable attention in the past decades, possibly, due to its intermediary role in the transport of arterial and venous blood. The flow has been studied in different perspectives. Some considered the nature of the vessels, and others the nature and physical composition of blood. [1] investigated analytically the flow in bifurcating porous channels the method of perturbation series solution, and noticed the existence of imaginary parts in the solutions but did not examine their relevance in terms of the oscillatory flow behaviours. Therefore, the aim of this study is to investigate the roles of the oscillatory flow characteristics in the flow of blood in bifurcating capillaries. Blood flows from the heart to the arteries through the influence of pressure waves and pulses produced in the heart. These pressure waves and pulses are damped out or seriously reduced when they reached the capillaries, and as such the pulsating or non-stationary part is neglected. However, due to the curved nature some secondary or oscillatory flow behaviours abound.

Blood capillaries are small vessels connecting the arteries to the veins so as to reach individual cells of the body for the purpose of exchange of materials. They constitute a network of branched and un-branched thin cylindrical porous tubes [2]. Each is about 5-10 microns (μ) in diameter [3]. The flow is slow [4] such that the Reynolds and Womersley numbers of the flow in the capillaries are less than one [5]. Moreover, capillaries are porous and this is said to dominate the elasticity of the walls such that the distensibility factor is neglected [6]. Thus they are treated as rigid vessels [7]. Similarly, it is suggested that blood in the capillaries seems to be less sensitive to vessel geometries, therefore, its flow could be treated as Poiseuille [8]. On this note, the flow in bifurcating capillaries approximates that in the upstream/mother channel. More so, the transport processes in the capillaries is assumed bidirectional under the influence of osmotic gradients involving concentration, temperature and convection [9].

Usually, blood flow in the capillaries is considered on the basis of homogeneity and non-homogeneity of blood. On the basis of homogeneity, both the blood fluid and corpuscles are assumed to be in the same phase such that they flow with a common velocity; for the non-homogeneous case, the blood fluid and corpuscles are taken to be in different phases and as such flow with different velocities. [4,7] have detailed description of the non-homogeneous blood flow in micro-channels/capillaries; [10] examined it as non-homogeneous and solved the governing Stokes' equations by numerical integration.

More so, on the homogeneous background, [11] and [12] considered blood flow in the capillaries and solved the governing Stokes' equations using series expansion method; [13] studied the blood flow and permeability in micro-vessels/capillaries but with emphasis on the mechanical aspect of the flow using *in vivo* and *in vitro* approach; [14] investigated the MHD oscillatory flow of blood in a capillary where thermal radiation and chemical reaction are present but with a non-Newtonian view using numerical approach, and obtained results for the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number when the parameters are varied; [15] examined the flow of blood in the capillaries using the method of perturbation series expansion, and observed that the increase in Grashof number increases the velocity, Nusselt and Sherwood numbers, whereas the increase in the Hartmann number decreases the velocity.

Additionally, some other literatures exist on related flows. For example, [16] studied the steady MHD incompressible viscous flow of a bio-fluid through a curved pipe of circular cross section using spectral method, and found that axial velocity increases as the Dean number increases, whereas it decreases as the curvature and magnetic parameters increase; [17] investigated steady MHD flow of a viscous incompressible fluid through a rotating curved pipe of circular cross-section using spectral approach, and noticed that for Taylor number greater than zero, the Coriolis force enforces the curvature effect, whereas for Taylor number less than zero, the Coriolis force exhibits an opposite effect to that of the curvature.

This paper examines the effects of chemical reaction rate, magnetic field force, heat exchange parameter and Grashof number on the concentration and velocity factors in the oscillatory blood flow in bifurcating capillaries.

2 Problem Formulation

This model is developed on the assumptions that the capillaries are porous; blood is incompressible, Newtonian and electromagnetic; the fluid velocity is symmetrical about the θ-axis. Then, for a two-dimensional situation, the equations describing the creepy flow, considering the Boussinesq approximations in the non-dimensionalized form (as developed from [18]) are:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\Re\partial w}{\partial x}=0 \tag{1}$$

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}=\frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial w}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial r}-\left(M^2+\chi^2\right)w=\Re\frac{\partial p}{\partial x} \\ -Gr\Theta-Gc\Phi \tag{3}$$

$$\frac{\partial^2\Theta}{\partial r^2}+\frac{1}{r}\frac{\partial\Theta}{\partial r}+N^2\Theta=Pe_h(u\frac{\partial\Theta}{\partial r}+\Re w\frac{\partial\Theta}{\partial x}) \tag{4}$$

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\delta_1^2\Phi=Pe_m(u\frac{\partial\Phi}{\partial r}+\Re w\frac{\partial\Phi}{\partial x}) \tag{5}$$

with the boundary conditions:

$$u=1,\quad w=1,\ \Theta=1,\ \Phi=1 \text{ at } \quad r=0 \tag{6}$$

$$u=0,\ w=0,\ \Theta=\Theta_w,\ \Phi=\Phi_w \text{ at } r=1 \tag{7}$$

where the dimensionalized terms are

$$r=\frac{r'}{R_o},\ x=\frac{\Re x'}{l},\ \Theta=\frac{T'-T_\infty}{T_w-T_\infty},\ \Phi=\frac{C'-C_\infty}{C_w-C_\infty},\ w=\frac{w'R_o}{l},\ p=\frac{\left(p'-p_\infty\right)r_o^3}{\rho l\upsilon},\ \mathrm{Re}=\upsilon l,$$
$$M^2=\frac{\sigma_e B_o^2}{\rho\mu\mu_m}\ N^2=\frac{Q}{k_o},\chi^2=\frac{R_o}{\kappa},\delta_1^2=\frac{k_r^2}{D},$$
$$Sc=\frac{\upsilon}{D},\mathrm{Pr}=\frac{\mu Cp}{k_o},Gr=\frac{g\beta_1\left(T_w-T_\infty\right)}{\upsilon^2},$$
$$Gc=\frac{g\beta_2\left(C_w-C_\infty\right)}{\upsilon^2},$$
$$Pe_m=\mathrm{Re}\,Sc\quad Pe_h=\mathrm{Re}\,\mathrm{Pr}\ ,M_1^2=M^2+\chi^2$$

and x and r are the axial and radial coordinates directions; u, w are the velocity vectors with respect to the orthogonal coordinate directions (r, x); ρ the fluid density; p the pressure; β_1 and β_2 are the volumetric expansion coefficient for temperature and concentration, respectively; Θ and Φ are the non-dimensionalized temperature and concentration; $\Re$ is the aspect ratio; υ is the kinematic viscosity; l is the characteristic length of the tube; μ the viscosity; μ_m the magnetic permeability of the fluid; **g** the gravitational field vector; κ is the permeability parameter of the porous medium; B_o is the applied uniform magnetic field strength due the nature of the blood and the earth field; σ_e is the electrical conductivity of the fluid; k_o the thermal conductivity; C_p the specific heat capacity at constant pressure; Q is the heat absorption coefficient; D the diffusion coefficient; k_r^2 is the rate of chemical reaction occurring in the system; R_o is the characteristic radius of the capillary; M^2 is the Hartmann number; Re is the Reynolds number; N^2 is the environmental temperature differential parameter, otherwise called the Heat exchange parameter; χ^2 is the porosity parameter; δ_1^2 is the chemical reaction parameter; Sc the Schmidt number; Pr the Prandtl number; (Gr,Gc) are the Grashof number due to temperature and concentration difference; (Pe_h, Pe_m) are the Peclet number due to heat and mass transfers.

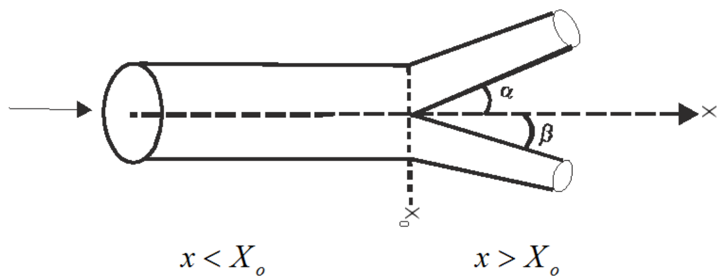

Fig. 1. A physical models of a bifurcating capillary (where α and β are the bifurcation angles)

A look at the flow structure in Fig. 1, shows that there are different flow pattern in the region before and after the nodal point. $x = X_o$. The region $x < X_o$ represents the upstream/mother channel, while the region $x > X_o$ represents the downstream/daughter channels. Therefore, the flow in the upstream terminates at $x = X_o$, while that of the daughter/downstream start at $x = X_o$. The flow in the mother/upstream is laminar and Poiseuille. The local stream-wise direction in the upstream channel is x, while that of the daughter/downstream is along the bifurcation angles α and β , off the x-axis. According to [8], the flow in the bifurcating blood capillaries approximates the Poiseuille flow. Therefore, the flow characteristics in the upstream channel shall be used to describe the flow in the capillaries

Equations (1)-(5) are nonlinear and coupled. Therefore, to obtain analytic solutions we seek for perturbation series solutions of the form

$$f(r,x) = f_o(r,x)) + \xi f_1(r,x) + \ldots \tag{8}$$

where $f_o(r,x)$ represents the flow variables in the upstream and $f_1(r,x)$ for those in the downstream, $\xi = \text{Re} < 1$ is the perturbation parameter which is extremely small. The choice of this perturbation parameter is based on the fact that, in the mother channel, that is, in the region $X < X_0$, the flow is laminar and Poiseuille. But as it flows towards the point of bifurcation, it experiences some forms of disturbances due to a change in the geometrical configuration such that its inertial force rises, and consequently the Reynolds number and momentum increase. Similarly, we assume the flow is fully developed such that $\frac{\partial u}{\partial r} = \frac{\partial p}{\partial r} = 0$; $f_0(r,x) = f_{00}(r) - \gamma x$; $p = \aleph x - \frac{\aleph x^2}{\Re}$ for the pressure in which $\aleph x$ is the pressure in the mother/upstream region (see [18]), then the equations governing the flow in this stream are as follows:

$$\frac{\partial^2 w_{00}}{\partial r^2} + \frac{1}{r}\frac{\partial w_{00}}{\partial r} - M_1^2 w_{00} = \frac{\Re \partial p_{00}}{\partial x}$$
$$- Gr\Theta_{00} - Gc\Phi_{00} \tag{9}$$

$$\frac{\partial^2 \Theta_{00}}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta_{00}}{\partial r} + N^2\Theta_{00} = -\gamma \Re Pe_h w_o \tag{10}$$

$$\frac{\partial^2 \Phi_{00}}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi_{00}}{\partial r} + \delta_1^2\Phi_{00} = -\gamma \Re Pe_m w_o \tag{11}$$

with the boundary conditions

$$w_{00} = 1,\ \Theta_{00} = 1,\ \Phi_{00} = 1 \text{ at } r = 0 \tag{12}$$

$$w_{00} = 0,\ \Theta_{00} = \Theta_w,\ \Phi_{00} = \Phi_w \text{ at } r = 1 \tag{13}$$

Now, we shall eliminate w_{00} from equations (10) and (11) by taking the

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - M_1^2\right)$$

of both sides of the equations to get

$$\left[\left(D_r - M_1^2\right)\left(D_r + N^2\right) - \gamma R\varepsilon\right]\Theta_{00} \qquad = \gamma R\varepsilon\Phi_{00} - \gamma\Re\varepsilon Pe_h\aleph \tag{14}$$

and

$$-\gamma\Re\varepsilon Pe_h\aleph + \gamma R\varepsilon\Theta_{00}$$
$$= \left[\left(D_r - M_1^2\right)\left(D_r + \delta_1^2\right) - \gamma R\varepsilon\right]\Phi_{00} \tag{15}$$

where

$$D_r = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) \text{ and } \varepsilon = Pe_h Gr = Pe_m Gc,\ Gr = Gc$$

Additionally, we shall eliminate Φ_{00} from equations (14) and (15) by taking

$$\left[\left(D_r - M_1^2\right)\left(D_r + \delta_1^2\right) - \gamma R\varepsilon\right]$$

of equation (14) and multiplying through equation (15) by $\gamma R\varepsilon$ so that on subtracting the first result from the second one, we have

$$\left[\left(D_r - M_1^2\right)\left(D_r + N^2\right) - \gamma R\varepsilon\right]\left[\left(D_r - M_1^2\right)\left(D_r + \delta_1^2\right) - \gamma R\varepsilon\right]\Theta_{00} = -\gamma R\varepsilon Pe_h \aleph + \ldots \tag{16}$$

More so, we shall eliminate Θ_{00} from equations (14) and (15) by multiplying through equation (14) by $\gamma R\varepsilon$ and taking

$$\left[\left(D_r - M_1^2\right)\left(D_r + N^2\right) - \gamma R\varepsilon\right]$$

of equation (15) so that on subtracting the first result from the second, and rearranging, we have

$$\left[\left(D_r - M_1^2\right)\left(D_r + N^2\right) - \gamma R\varepsilon\right]\left[\left(D_r - M_1^2\right)\left(D_r + \delta_1^2\right) - \gamma R\varepsilon\right]\Phi_{00} = -\gamma R\varepsilon Pe_h \aleph \tag{17}$$

A comparison of equation (16) with equation (17) shows that they the same. Therefore, $\Phi_{00} = \Theta_{00}$, and as such their solutions are the same.

Expanding equation (17), gives a fourth order characteristic equation, which when split into two tractable parts (even and odd power terms) and solving the even power term, we have

$$\Theta_{00}(r) = P_1 I_o\left(\lambda_8^{1/2} r\right) + \frac{I_o\left(\lambda_8^{1/2} r\right)}{2}\lambda_8^{1/2} Q_1 \frac{r I_1\left(\lambda_6^{1/2} r\right)}{\lambda_6^{1/2}} \tag{18}$$

Furthermore, assuming $\Pr Gr = \Pr Gc$ and $\Phi_{oo} = \Theta_{oo}$, substituting Φ_{oo} and Θ_{oo} in eq. (9) and solving, we have

$$\begin{aligned} w_{oo}(r) &= V_1 I_o(M_1 r) \\ &- M_1 r I_o(M_1 r)\left[-\frac{\Re\aleph r}{2} + GrD_1 \frac{I_1(\lambda_8^{1/2} r)}{\lambda_6^{1/2}} \right. \\ &\left. - \frac{r^5}{20} - \frac{\lambda_6 r^7}{114} + M_1^2\left(\frac{r^4}{16} + \frac{\lambda_6 r^6}{192}\right)\right] \\ &\left(-\frac{M_1 r}{2} + \frac{M_1^2 r^3}{4}\right)\left[\frac{\Re\aleph I_1(M_1 r)}{M_1}\right. \\ &- 2GrD_1\left(\frac{I_1(\lambda_8^{1/2} r)}{\lambda_8^{1/2}} - \frac{M_1^2 r^3}{6} - \frac{M_1^2 \lambda_8 r^5}{40}\right) \end{aligned}$$

$$+\frac{r^3}{6}+\frac{\lambda_6 r^5}{80}+M_1^2\left(\frac{r^4}{16}+\frac{\lambda_6 r^6}{192}\right)\Bigg]\Bigg] \quad (19)$$

3 Results and Discussion

We used Maple 18 for our computations. For constant values of Pr =0.71, Re=0.03; γ_1 = 0.6, γ_2 =0.6, γ =0.7, Φ_w = 2.0, Θ_w =2.0, $\Re = 0.8$, and varied values of δ_1^2 =0.1, 0.3, 0.5. 1.0, 5.0; N^2 = 0.1, 0.3, 0.5. 1.0, 5.0; M^2 =0.1, 0.3, 0.5. 1.0, 5.0; Gr=0.1, 0.3, 0.5. 1.0, 5.0; we have the results shown in Tables 1–4. These tables show that the flow velocity increases as the rate of chemical reaction, Hartmann number (M^2≤I.0), Heat exchange parameter and Grashof number (Gr/Gc≤I.0) respectively, increase.

Table 1. Velocity-chemical reaction rate

r	δ_1^2 =0.1	δ_1^2 =0.3	δ_1^2 =0.5	δ_1^2 =1.0	δ_1^2 =5.0
0.0	-0.36328199I	0.359416906I	0.368320711I	0.401327653I	0.464588355I
0.2	-0.35870131I	0.3528408142I	0.3612898353I	0.3930844632I	0.4571474604I
0.4	-0.34592389I	0.3340905770I	0.3412147251I	0.3694318709I	0.4351587332I
0.6	-0.32774512I	0.3059649607I	0.3109846512I	0.3333123769I	0.3990192654I
0.8	-0.30845479I	0.2726520016I	0.2748315768I	0.2885693688I	0.3470627970I
1.0	-0.29312320I	0.2389109513I	0.2402955081I	0.2419522519I	0.2708109823I

Table 2. Velocity-Hartmann number

r	M^2=0.1	M^2=0.3	M^2=0.5	M^2=1.0	M^2=5.0
0.0	0.2639014849I	0.3114408964I	0.3876844866I	0.5736734869I	-24.6222126I
0.2	0.2583992618I	0.3051927331I	0.3805461903I	0.5668512426I	-30.7835927I
0.4	0.2423136801I	0.2870988249I	0.3603153536I	0.5506124969I	-49.1710680I
0.6	0.2168696658I	0.2590327883I	0.3303430146I	0.5359877494I	-89.0313150I
0.8	0.1839511642I	0.2238223629I	0.2955092709I	0.5359558022I	-197.568668I
1.0	0.1457560168I	0.1846758476I	0.2610195833I	0.5575863648I	-507.656414I

Table 3. Velocity-Heat exchange parameter

r	N^2=0.1	N^2 =0.3	N^2 =0.5	N^2 =1.0	N^2 =5.0
0.0	0.3628856652I	0.3641383645I	0.3668353860I	0.393909983I	0.4530320097I
0.2	0.3560461385I	0.3572654254I	0.3598811703I	0.385670012I	0.4486760210I
0.4	0.3365248924I	0.3376482990I	0.3400298217I	0.362015322I	0.4206821694I
0.6	0.3071632297I	0.3081392636I	0.3101571606I	0.325859781I	0.3807568522I
0.8	0.2721612063I	0.2729486939I	0.2744926468I	0.281025050I	0.3231145097I
1.0	0.2361398399I	0.2366912558I	0.2376250114I	0.238000235I	0.2386197556I

Table 4. Velocity-Grashof number

r	Gr=0.1	Gr=0.3	Gr=0.5	Gr=1.0	Gr=5.0
0.0	0.0717144449I	0.218353646I	0.365958128I	0.734719241I	-3.84847884I
0.2	0.0703950464I	0.2142558418I	0.3590318833I	0.7207126373I	-3.80598853I
0.4	0.0666345914I	0.2025636709I	0.3392612883I	0.6807168104I	-3.68842974I
0.6	0.0610033144I	0.1849936546I	0.3095145347I	0.6204729370I	-3.52425613I
0.8	0.0543417285I	0.1640777869I	0.2740154215I	0.5484227820I	-3.35655423I
1.0	0.0474648952I	0.1425222205I	0.2373631977 I	0.4739117773I	-3.23627999I

Blood released from the heart into the arteries and capillaries mainly constitutes water, digested food materials, mineral salts and possibly, drugs taken. The chemical content may spark-up a chemical reaction (assumed an order one reaction), which leads to the required depletion of the chemicals in the system. It may be exothermic or endothermic, implying that heat is given out or absorbed. The depletion of the chemical content leads to an increase in concentration of the transported blood (see Table 1). The increase in the concentration of the blood makes it osmotically higher than that in the tissues, thus making its diffusion possible.

Blood is saline or slightly acidic in nature; therefore, it is electrolytic and magnetically susceptible. The chemical content exists as ions or charges. The motion of these ions in the presence of the Earth magnetic field produces electric currents. In addition, the action of the magnetic field on the currents generates a mechanical force, the Lorentz force, which modifies the flow. The Lorentz force tends to fractionalize and polarize the fluid chemical contents such that they cluster around the magnetic field. Similarly, the freezing of the velocity makes the fluid to be concentrated in the region. This may accounts for what is seen in Table 1.

Even so, the analysis shows that any increase in the Hartmann number in the range of $0.1 \leq M^2 \leq I.0$ tends to increase the velocity whereas it decreases for $M^2 \geq 5.0$ (see Table 2). In many flow problems, the Hartmann number is known to freeze up the velocity field. The re-ordering of the flow in the range of $0.1 \leq M^2 \leq I.0$, here, could be due to the oscillatory effect.

Furthermore, the heat exchange parameter depends on the external/environmental temperature, which in particular, depends on the radiation from the sun. It rises when the temperature is high and vice versa. The increase in the temperature invariably leads to an increase in the fluid velocity. More so, the increase in the temperature increases the permeability of the cell walls on one hand, and reduces the fluid viscosity on the other hand. This tends to enhance the flow velocity (see Table 3).

Similarly, in systems involving heat flow, heat is either generated or absorbed. In whichever way, the existence of the heat looses the fluid particles from the grip of viscosity and grants them buoyancy. The energization of the fluid particles tends to increase the velocity structures, as seen in Table 4.

The increase and decrease in the temperature, concentration and velocity has some attendant health implications in man (for example). The increase in these flow variables combiningly increases the rate of transport of blood rich in oxygen and nutrients in the capillaries, thus increasing the availability of such blood in the tissues. These enhance the physiological well-being of man.

4 Conclusions

We investigated the oscillatory flow of blood in bifurcating fine capillaries. The results show that the increase in the rate of chemical reaction, Hartmann number ($M^2 \leq I.0$), heat exchange parameter and Grashof number ($Gr \leq I.0$) tends to increase the velocity structure of the flow. These results have attendant implications. They tend to increase the rate of transport of oxygen and nutrients–rich blood in the capillaries, and their availability to the tissues. In fact, these enhance the physiological well-being of man.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Okuyade WIA. Magneto-hydrodynamic (MHD) fluid flow in bifurcating porous channels, A Ph.D Dissertation, Department of Mathematics/Statistics, University of Port Harcourt, Nigeria; 2015 (Unpublished).

[2] Lipowsky HH. Mechanics of blood flow in microcirculation. In handbook of bio-engineering (ed. Skalak, R. and Chien, S.), McGraw-Hills, New York. 1987;18:1-18.

[3] Skalak Richard, Skalak Thomas, Nihat Ozkaya. Bio-fluid mechanics rev. Fluid Mechanics. 1989;21: 167–204.

[4] Lighthill MJ. Mathematical bio-fluid dynamics. Society for Industrial and Appl. Maths, Philadelphia, Pennsylvania. 1972;19103:199–281.

[5] Wilmer WN, Michael, Rourke EO. McDonald's blood flowin arteries, Edward Arnold, London; 1980.

[6] Petrilan Sonia, Albert Balazs. Mathematical model of blood flow in capillaries. J. Eng, Tome VIII, Fascicula 3. 2009;352-359.

[7] Ducharmel Robert, Kapadia Phiroze, Dowden John. A mathematical model of the flow of blood cells in fine capillaries. J. Biomech. 1991;24(5):299–306.

[8] Lighthil MJl. Physiological fluid dynamics: A survey. J. Fluid Mech. 1971;52:475–497.

[9] Guyton AC. Textbook of medical physiology, W.D. Saunders, London. 1966.

[10] Lin KL, Lopez L, Hellums JD. Blood flow in capillaries. Microvasc. Res. 1973;5:7-19.

[11] Whitmore, Raymond L. A theory of blood flow in small vessels. J. Appl. Physiol. 1967;22(4):767–771.

[12] Chien S, Usami S, Skalak R. Blood flow in small tubes. In handbook of physiology: The cardiovascular system iv (ed. Renkin EM., Michel CC).Md. Am. Physiol. Society, Bethesola. 1984;217-249.

[13] Masako Sugihara-Seki, Bingmei M Fu. Blood flow and permeability in micro-vessels. Fluid Dyn. Res. 2005;37(1-2).
DOI: 10.1016/j.fluiddyn.2004.03.006

[14] Sinha A, Shit GC. Oscillatory blood flow through a capillary in presence of thermal radiation. Int. J. Biomath. 2015;08(1550014).
DOI: 10.1142/S179352451550014X

[15] Okuyade WIA. MHD blood flow in bifurcating porous fine capillaries. African J Sci. Res. 2015;4(4): 56-59.

[16] Hoque M, Alam M, Mohammad Ferdows, Osman A Be´g. Numerical simulation of Dean number and curvature effects on magneto-bio-fluid flow through a curved conduit. Proc. Inst. MechE Part H: J Engineering in Medicine. 2013;227(11):1155–1170.
DOI: 10.1177/0954411913493844

[17] Mainul Hoque, Mahmud Alam. A numerical study of MHD laminar flow in a rotating curved pipe with circular cross section. Open J. Fluid Dyn. 2015;5:121-127.
DOI: 10.4236/ojfd.2015.52014

[18] Bestman AR. Global models of the biomechanics of green plants, part 1. Int. J. Energy Res. 1991;16: 677–684.

Appendices

$$P_1 = \frac{1}{I_o\left(\lambda_8^{1/2}\right)}\left[\Theta_w - \frac{I_o\left(\lambda_8^{1/2} r\right)}{2}\lambda_8^{1/2} Q_1 \frac{I_1\left(\lambda_6^{1/2}\right)}{\lambda_6^{1/2}}\right]$$

$$Q_1 = \frac{1}{I_o\left(\lambda_6^{1/2}\right)}\left[\Theta_w - \frac{I_o\left(\lambda_6^{1/2}\right)}{2}\lambda_6^{1/2} R_1 \frac{J_1\left(\lambda_4^{1/2}\right)}{\lambda_4^{1/2}}\right]$$

$$R_1 = \frac{1}{J_o\left(\lambda_4^{1/2}\right)}\left[\Theta_w - J_o\left(\lambda_4^{1/2}\right)\lambda_4^{1/2} S_1 \frac{J_1\left(\lambda_2^{1/2}\right)}{\lambda_2^{1/2}}\right]$$

$$S_1 = \frac{1}{J_o\left(\lambda_2^{1/2}\right)}\left[\Theta_w - J_o\left(\lambda_2^{1/2}\right)\frac{\pi\lambda_2^{1/2}}{2}\gamma\Re\varepsilon Pe_h \aleph\right]$$

$$\lambda_2 = \sqrt{\left\{\frac{1}{2}\left(-\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right) + \right.\right.}$$
$$\left.\left.\sqrt{\left(\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right)^2 + 4\left(-M_1^2\delta_1^2\gamma\Re\varepsilon + \gamma^2\Re^2\varepsilon^2\right)\right)}\right)\right\}$$

$$\lambda_4 = \sqrt{\left\{\frac{1}{2}\left(-\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right) - \right.\right.}$$
$$\left.\left.\sqrt{\left(\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right)^2 + 4\left(-M_1^2\delta_1^2\gamma\Re\varepsilon + \gamma^2\Re^2\varepsilon^2\right)\right)}\right)\right\}$$

$$\lambda_6 = i\sqrt{\left\{\frac{1}{2}\left(-\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right) + \right.\right.}$$
$$\left.\left.\sqrt{\left(\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right)^2 + 4\left(-M_1^2\delta_1^2\gamma\Re\varepsilon + \gamma^2\Re^2\varepsilon^2\right)\right)}\right)\right\}$$

$$\lambda_8 = i\sqrt{\left\{\frac{1}{2}\left(-\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right) - \right.\right.}$$
$$\left.\left.\sqrt{\left(\left(-M_1^2 + 2N^2M_1^2 + 2M_1^2 - N^2\delta_1^2 - M_1^4 - 2\gamma R\varepsilon\right)^2 + 4\left(-M_1^2\delta_1^2\gamma\Re\varepsilon + \gamma^2\Re^2\varepsilon^2\right)\right)}\right)\right\}$$

$$T_1 = \frac{1}{I_o\left(\lambda_{16}^{1/2}\Re\alpha x\right)}\left[\gamma_1\Theta_w - \frac{I_o\left(\lambda_{16}^{1/2}\Re\alpha x\right)}{2}\lambda_{16}^{1/2} U_1 \Re\alpha x \frac{I_o\left(\lambda_{14}^{1/2}\Re\alpha x\right)}{\lambda_{14}^{1/2}}\right]$$

$$U_1 = \frac{1}{I_o\left(\lambda_{14}^{1/2}\Re\alpha x\right)}\left[\gamma_1\Theta_w - \frac{I_o\left(\lambda_{14}^{1/2}\Re\alpha x\right)}{2}\lambda V_1 \Re\alpha x \frac{J_1\left(\lambda_{12}^{1/2}\Re\alpha x\right)}{\lambda_{12}^{1/2}}\right]$$

$$V_1 = \frac{1}{J_o\left(\lambda_{12}^{1/2}\Re\alpha x\right)}\left[\gamma_1\Theta_w - \frac{J_o\left(\lambda_{12}^{1/2}\Re\alpha x\right)}{2}\pi\lambda_{12}^{1/2} W_1 \Re\alpha x \frac{J_1\left(\lambda_{10}^{1/2}\Re\alpha x\right)}{\lambda_{10}^{1/2}}\right]$$

$$W_1 = \frac{1}{J_o\left(\lambda_{10}^{1/2}\Re\alpha x\right)}\left[\gamma_1\Theta_w - \frac{J_o\left(\lambda_{10}^{1/2}\Re\alpha x\right)}{2}\pi\lambda_{10}^{1/2}\left[-\gamma^2\Re^2\varepsilon^2 Pe_h\left(\aleph + \aleph_1 x\right)\frac{\left(\Re\alpha x\right)^2}{2}\right.\right.$$
$$\left.\left. + 2\gamma^2\Re^2\varepsilon^2 D_1 \Re\alpha x \frac{I_1\left(\lambda_6^{1/2}\Re\alpha x\right)}{\lambda_6^{1/2}}\right]\right]$$

$$\lambda_{10} = \sqrt{\frac{1}{2}\{-\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right) +}$$
$$\sqrt{\left(\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right)^2 - 4\left(\gamma \Re\varepsilon\left(M_1^2 N^2 \delta_1^2 - M_1^2 N^2 - M_1^2 \delta_1^2\right)\right)\right)}\}$$
$$\lambda_{12} = \sqrt{\frac{1}{2}\{-\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right) -}$$
$$\sqrt{\left(\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right)^2 - 4\left(\gamma \Re\varepsilon\left(M_1^2 N^2 \delta_1^2 - M_1^2 N^2 - M_1^2 \delta_1^2\right)\right)\right)}\}$$
$$\lambda_{14} = i\sqrt{\sqrt{\frac{1}{2}\{-\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right) +}}$$
$$\sqrt{\left(\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right)^2 - 4\left(\gamma \Re\varepsilon\left(M_1^2 N^2 \delta_1^2 - M_1^2 N^2 - M_1^2 \delta_1^2\right)\right)\right)}\}$$
$$\lambda_{16} = i\sqrt{\sqrt{\frac{1}{2}\{-\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right) -}}$$
$$\sqrt{\left(\left(-\delta_1^2 M_1^2 - \left(N^2 - M^2\right)\left(\delta_1^2 - M^2\right) - M_1^2 N^2\right)^2}$$
$$-4\left(\gamma \Re\varepsilon\left(M_1^2 N^2 \delta_1^2 - M_1^2 N^2 - M_1^2 \delta_1^2\right)\right)\Big)\}$$

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/19866