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Topologized Cut Vertex and Edge Deletion

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper discussed and study a new result of non-topologized graph by using cut vertex and cut edge component of the graph makes the graph to be Topologized graph. This concept implemented to some families of graph.

Keywords: Topology; labeling graph; prism graph; wheel graph.

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1 Introduction

The topologized graph was introduced by Antoine Vella [1]. Topological graph theory is a branch of graph theory. It studies the embedding of graphs in surfaces, spatial embeddings of graphs, and graphs as topological spaces. It also studies immersions of graphs. The Introduce this concept is a graph with cut vertex of components are topologized. In every vertex and edge boundary [2] are n = 0, 1, 2 a topologized graph. If the every path and circuit is topologized graph but a non topologized graph use to some definition

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that graph any single component is topologized. The non topologized graph components are topologized graph.

2 Preliminaries

2.1 Topological space

A topology on a set X is a collection τ of a subset of X with the following properties:

- Φ and X are in τ.
- The union of the element of any subcollection of τ is in τ (arbitrary union).
- The intersection of the element of any finite subcollection of τ is in τ.

The set X for which a topology τ has been specified is called a topological space [3].

2.2 Topologized graph

A topologized graph is a topological space X such that

- Every singleton is open or closed.
- $\forall\, x \in X$, $|\partial(x)| \leq 2$, where $\partial(x)$ is the boundary of a point x [1].

2.3 Cut vertex

Let G be a connected graph .let v be a vertex of G. Then v is a cut vertex of G if and only if the vertex deletion G-v is a vertex cut of G. That is such that v is disconnected. Thus a cut vertex of G is a singleton vertex cut of v [4].

2.4 Cut edge

Let G be a connected graph .let e be a edge of G. Then e is a cut edge of G if and only if the edge deletion G-e is a edge cut of G. That is such that e is disconnected. Thus a cut edge of G is a singleton edge cut of v [5].

2.5 Vertex deletion

Let G = (V, E) be an (undirected) graph. let $W \subseteq V$ be a set of vertices of G. Then the graph obtained by deleting W from G denoted by G – W, is the subgraph induced by V/W. The vertex deletion separates the graph into disconnected component [2].

2.6 Edge deletion

Let G = (V, E) be an (undirected) graph. let $F \subseteq V$ be a set of vertices of G. Then the graph obtained by deleting F from G denoted by G – F, is the subgraph induced by V/W.The edge deletion separates the graph into disconnected component [5].

2.7 Tree

A tree is a connected graph that contains no cycle. In a tree, every pair of points is connected by a unique path [6].

2.8 Spanning tree

A spanning tree is a subset of graph G. which has all the vertices covered with minimum possible number of edges [7].

2.9 Fundamental circuit

A graph is formed by vertices and edges connecting the vertices. Example: formally, a graph is a pair of sets (V, E) where v is the set of vertices and E is the set of edges formed by pairs of vertices [8].

2.10 Prism graph

A prism graph is a graph that has one of the prisms as its skeleton [9].

2.11 Wheel graph

A wheel graph is a graph by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices can also be defined as the 1- skeleton of an (n_1) - gonal pyramid [10].

2.12 Labeling graph

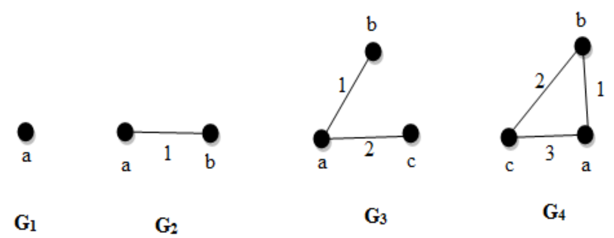
A graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph. Likewise, an edge labeling is a function of E to a set of labels [11].

3 Main Results

Example 1

The three points (n=3) tree and circuit is a topologized graph.

The following graphs are n = 0, 1, 2, 3



G_1 graph boundary is (v, e both are) empty

Since, G_1 graph is topologized graph.

G_2 graph boundaries

$\partial\,|1| = \{a, b\} = 2$
$\partial\,|a| = 1$
$\partial\,|b| = 1$

G_2 graph boundary is 1 or 2, for $n \leq 2$

ie, G_2 graph is topologized.

G_3 graph boundaries

$\partial\ |1| = \{a, b\} = 2$
$\partial\ |2| = \{a, c\} = 2$
$\partial\ |a| = \{1, 2\} = 2$
$\partial\ |b| = 1$
$\partial\ |c| = 1$

G_3 graph is topologized for $n \leq 2$

G_4 graph is also topologized. Hence the tree and circuit n=3 is a topologized graph.

Theorem 3.1

In a Tree with cut vertex the components are topologized graph.

Proof:

Let Tree be the connected graph that contains no cycle.

By the definition of vertex split into the more vertices is called cut vertex.

Take one Tree,

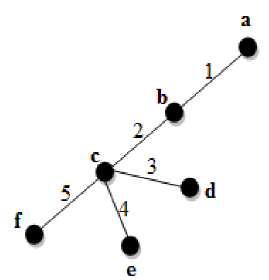


Fig. 1. Tree with six vertices

A Tree with cut vertex split into two components (**C_1** & **C_2**).

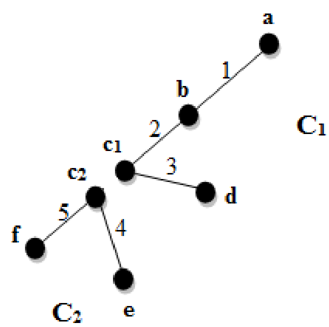


Fig. 2. t with Cut vertex

A vertex C split into the two vertices c_1 and c_2.

The set of components {a, 1, b, 2, c_2, 3, d} and {e, 4, c_2, 5, f} are topologized graph. if the C_1 and C_2 two components are topologized.

Hence the components are topologized graph.

Example 2

A cut vertex of spanning tree and then the components are topologized graph.

Theorem 3.2

In a spanning tree with four vertices all are not topologized. Then the all components are topologized.

Proof:

Let a spanning tree with three points. It is topologized.

The spanning tree with four points and all spanning tree is not topologized.

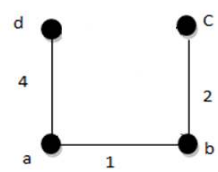


Fig. 3. S_1 Spanning tree

The above spanning tree is topologized for $n \leq 2$.

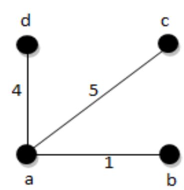


Fig. 4. S_2 Spanning tree

The above spanning tree is not topologized for $n \leq 3$

Boundary is ∂ (c) = {1, 5, 4} = 3

ie, A spanning tree is not topologized.

But, the above spanning tree with cut vertex that tree is topologized.

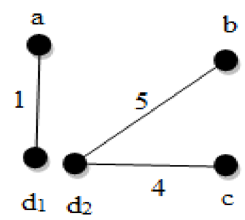


Fig. 5. S_2 Spanning tree with cut vertex

C_1 component boundaries are

∂ |a_1| = 4 = 1
∂ |d| = 4 = 1
∂ |4| = {a, d} = 2

C_1 component is topologized for n $\leq$ 2

The spanning tree components are topologized.

Otherwise that spanning tree is not topologized.

C_2 component boundaries are

∂ |c| = 5 = 1
∂ |5| = {a_2, c} = 2
∂ |a_2|= {5, 1} = 2
∂ |1| = {a_2, b} = 2
∂ |b| = 1 = 1

C_2 component is topologized, for n $\leq$ 2

C_1& C_2 components are topologized graph.

Hence the n points spanning tree with cut vertex if all the components are topologized graph.

Theorem 3.3

In a W_5 graph, then spanning tree with fundamental circuit is not topologized.

Proof:

Let be the some spanning tree is topologized

To prove that, some spanning tree with fundamental circuit is not topologized.

Let us take a wheel graph,

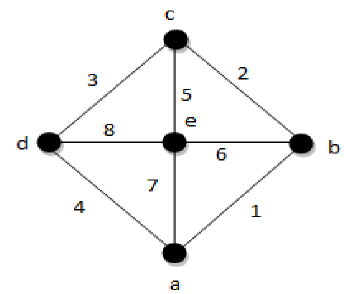
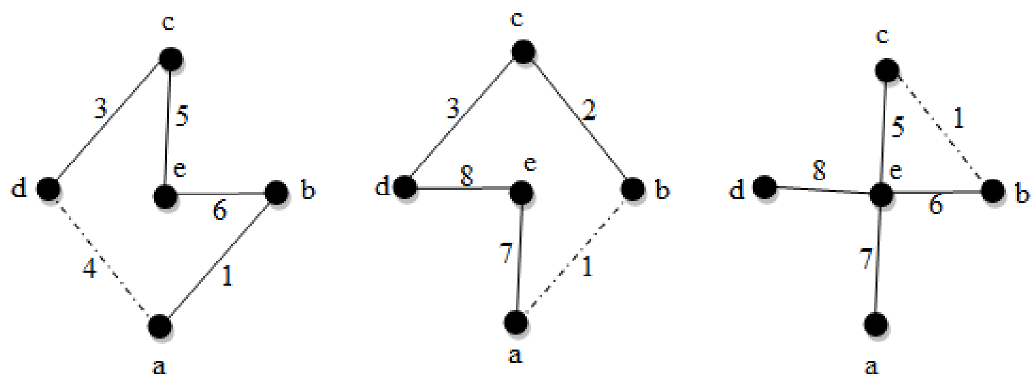


Fig. 6. W_5 graph

Some spanning trees with fundamental circuits are



S_1 **S_2** **S_3**

Fig. 7. Spanning tree with fundamental circuit of W_5 graph

Add one chord in a spanning tree formed by fundamental circuit.

The above $S_{1,}$ $S_{2,}$ S_3 spanning tree are form a fundamental circuit. But, all the spanning tree and fundamental circuit is not topologized.

If ∂ |x| ≤ 2

S_1 spanning tree is topologized. Then, form a fundamental circuit is also topologized graph.

Similar way, S_2 spanning tree is topologized.

If not, ∂ |x| ≤ 2

A S_3 spanning tree boundary is ∂ |x| = {5, 6, 7, 8}. Since if condition is does not satisfied.

Then the spanning tree S_3 not topologized graph.

But, a spanning tree with cut vertex and the components are topologized.

Then, S_3 form a fundamental circuit is not topologized.

S_3 boundaries are

$\partial\,\lvert 5\rvert = \{c, e\} = 2$	$\partial\,\lvert a\rvert = 7 = 1$
$\partial\,\lvert 6\rvert = \{e, b\} = 2$	$\partial\,\lvert b\rvert = \{6, 2\} = 2$
$\partial\,\lvert 7\rvert = \{e, a\} = 2$	$\partial\,\lvert c\rvert = \{5, 2\} = 2$
$\partial\,\lvert 8\rvert = \{e, d\} = 2$	$\partial\,\lvert d\rvert = 8 = 1$
$\partial\,\lvert 2\rvert = \{b, c\} = 2$	$\partial\,\lvert e\rvert = \{5, 6, 7, 8\} = 2$

Since, S_3 is not topologized.

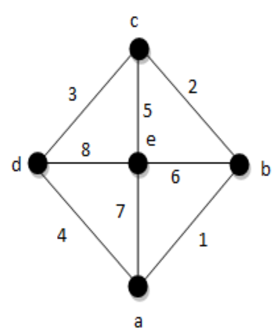
Hence the spanning tree with fundamental circuit is topologized or not topologized graph.

Remark

In every graph take the more spanning tree but all are not tpologized graph. atleast one spanning tree and spanning tree with fundamental circuit is topologized. Otherwise using cut vertex definition a spanning tree is topologized.

3.1 Wheel and prism graph with Cut vertex

Let us take a W_5 graph



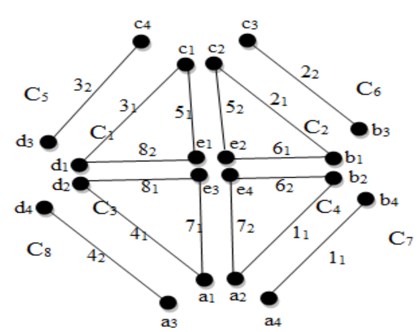


Fig. 8. W_5 graph **W_5 with cut vertex and cut edge**

The W_5 graph components are

$C_1 = \{e_1, 5_1, c_1, 3_1, d_1, 8_1\}$	$C_5 = \{d_3, 3_2, c_4\}$
$C_2 = \{e_2, 6_1, b_1, 2_1, c_2, 5_2\}$	$C_6 = \{c_3, 2_2, b_3\}$
$C_3 = \{a_1, 7_1, e_3, 8_1, d_2, 4_1\}$	$C_7 = \{b_4, 1_1, a_4\}$
$C_4 = \{a_2, 1_2, b_2, 6_2, e_4, 7_2\}$	$C_8 = \{a_3, 4_2, d_4\}$

Take one component C_1 take another component C_5

$C_1 = \{e_1, 5_1, c_1, 3_1, d_1, 8_1\}$ $C_5 = \{d_3, 3_2, c_4\}$

The boundaries are The boundaries are

$\partial\,\lvert e_1\rvert = \{8_2, 5_1\} = 2$	$\partial\,\lvert 5_1\rvert = \{e_1, c_1\} = 2$	$\partial\,\lvert d_3\rvert = 3_2 = 1$
$\partial\,\lvert c_1\rvert = \{5_1, 3_1\} = 2$	$\partial\,\lvert 3_1\rvert = \{c_1, d_1\} = 2$	$\partial\,\lvert c_4\rvert = 3_2 = 1$
$\partial\,\lvert d_1\rvert = \{3_1, 8_2\} = 2$	$\partial\,\lvert 8_2\rvert = \{d_1, e_1\} = 2$	$\partial\,\lvert 3_2\rvert = \{d_3, c_4\} = 2$

C_1 & C_5 components boundary is one or two is a topologized graph.

Since C_1 & C_5 components are topologized graph.Similarly $C_2, C_3, C_4, C_6, C_7, C_8$ are topologized graph.

Hence all the components are topologized graph.

Let us take Y_6 prism graph

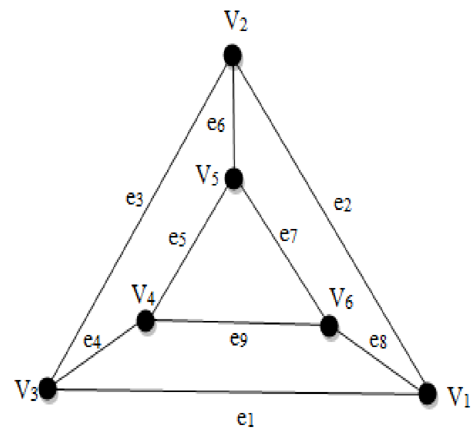


Fig. 9. Prism Y_6 graph

A Cut vertex split into the more vertices and also a cut edge split into the more vertices in the graph.

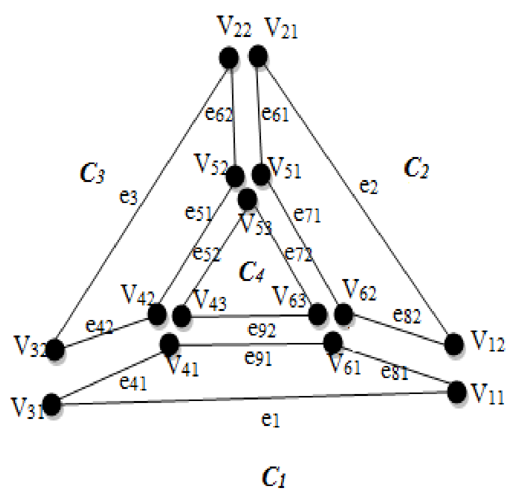


Fig. 10. Y_6 graph with cut vertex and cut edge

The above prism graph with cut vertex and inside cut edge split into more components for any component is topologized graph.

The components are

C_1= {$v_{11}, e_{81}, v_{61}, e_{91}, v_{41}, e_{41}, v_{31}, e_1$}
C_2 = {$v_{12}, e_{82}, v_{62}, e_{71}, v_{51}, e_{61}, v_{21}, e_2$}
C_3= {$v_{22}, e_{62}, v_{52}, e_{51}, v_{42}, e_{42}, v_{32}, e_3$}
C_4= {$v_{43}, e_{52}, v_{53}, e_{72}, e_{92}$}

Take one component ***C_4***

The boundaries are

∂ | v_{43}| = {e_{52}, e_{92}} = 2
∂ | e_{52}| = {v_{43}, v_{53}} = 2
∂ | v_{53}| = {e_{52}, e_{72}} = 2
∂ | e_{72}| = {v_{53}, v_{63}} = 2
∂ | v_{63}| = {e_{72}, e_{92}} = 2
∂ |e_{92}| = {v_{63}, v_{43}} = 2

Other all components also a circuit and the components every single vertex and edge boundary is 2. If it is a four components are topologized graph.

If the merged of four components are Y_6 and W_5 graph.

Since wheel and prism graph with cut vertex and cut edge then the components are topologized graph.

Theorem 3.4

Let G be a wheel graph. If any vertex and an edge of G are the cut points, then the disconnected components of the graph are topologized graph.

Theorem 3.5

Let G be a prism graph. If any vertex and an edge of G are the cut points, then the disconnected components of the graph are topologized graph.

3.2 Prism graph with vertex and edge deletion

Take a Fig. 9 Y_6 graph

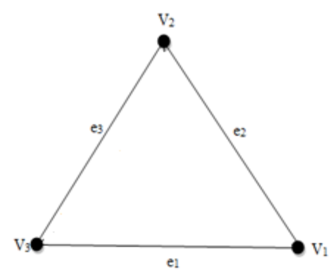


Fig. 11. Vertex deletions of prism (Y_6) graph

Vertex deletions of centre vertices are {v_4, v_5, v_6}. A set {v_1, e_2, v_2, e_3, v_3, e_1} of graph is topologized. The vertices are {v_1, v_2, v_3} are formed by one circuit is a topologized graph.

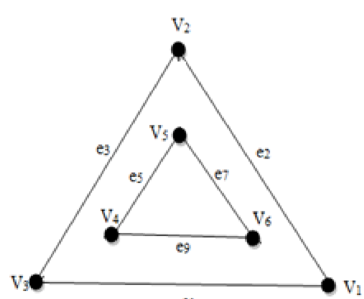


Fig. 12. Edge deletions of prism (Y_6) graph

Edge deletions of edges are {e_4, e_6, e_8}. A graph is separated into the two set of graphs {v_1, e_2, v_2, e_3, v_3, e_1} and {v_4, e_5, v_5, e_7, v_6, e_9} are topologized. The two set of graph is two different components.

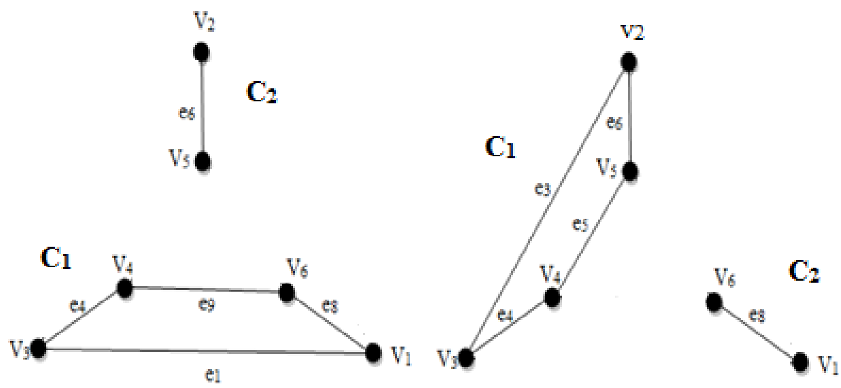


Fig. 13. Cut sets of prism (Y_6) graph

The cut sets are {e_3, e_5, e_7, e_2} and {e_2, e_7, e_9, e_1}.

A cut set of some edges a graph is disconnected into the two components. It is topologized graph. If the prism graph using cut set with four edges in the two components are topologized. Otherwise graph is disconnected but the components are not topologized.

4 Conclusion

In this paper, a new result of non-topologized graph by using cut vertex and cut edge deletion in every component of the graph makes to be Topologized. Then the graph is introduced and investigated and this concept implemented to the family of trees, wheel graph and prism graph. Further it may apply to the other family of graphs too.

Competing Interests

Authors have declared that no competing interests exist.

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