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Chaotic Graph on the Sphere

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Abstract

Our aim in the present article is to introduce and study a new type of chaotic graphs, namely chaotic graph on a sphere .We describe them by using chaotic matrices. This article introduces some operations on the chaotic graphs such as union and intersection; also both of the chaotic incidence matrices and the chaotic adjacency matrices representing the chaotic graphs induced from these operations will be introduced. Theorems governing these studies are obtained. Some applications on chaotic graphs are given.

Keywords: Chaotic; graph; sphere.

1 Introduction

Scientists of all fields have, for centuries, been searching for a simple theory that links all the systems of the Universe a "Theory of everything", a simple way to describe almost anything. In recent times, scientists have found a ray of hope: Chaos theory. It has brought scientists from many fields together to study this new emergency in theoretical science. Many scientists, who had been worried by the increasing specialization in science, are now breathing sighs of relief, as this theory unites scientists from fields as biology and mathematics. It does seems as though the world of science has found its "Theory of everything " [1,2].

At first, many scientists believed that the disorder in nature was not a legitimate problem, that things such as laser fusion demanded their immediate attention. Other scientists, with an eye for pattern, especially pattern that appeared on different levels at the same time, studied this irregularity, and tried to understand it. What they came up with was chaos theory. Chaos theory is where classical science has always dealt with predictability, and predictable elements of problem. It has always strayed a way from that which did not fit

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into the patterns that have been speculated in the past. Chaos theory can be applied in many scientific disciplines: mathematics, programming, microbiology, biology, computer, science, economics, engineering, finance, physics, population dynamics and others.

Chaotic behavior is observed in the laboratory of lasers, oscillating chemical reactions, magneto-mechanical devices and computer models of chaotic processes. We observe the chaotic behavior in nature include changes in weather, population growth in ecology, the dynamics of the action potentials in neurons, and molecular vibrations. There is some controversy over the existence of chaotic dynamics in plate tectonics and in economics. Most of successful applications of chaos theory has been done in ecology, where dynamical systems have been used to show how population growth can lead to chaotic dynamics. Currently chaos theory is applied to medical studies of epilepsy. One of the related field of physics is called quantum chaos theory. It investigates the relationship between chaos and quantum mechanics. The classical mechanics is a special case of quantum mechanics. If quantum mechanics does not demonstrate an exponential sensitivity to initial conditions, it is unclear how exponential sensitivity to initial conditions can arise in practice in classical chaos. Recently, the field, of relativistic chaos, has described systems that follow the laws of general relativity. The initial conditions of three or more bodies interacting through gravitational attraction (see the n-body problem) can be arranged to produce chaotic motion [3,4,5].

2 Definitions and Background

Basic topological and geometric concepts and definitions relevant to this article are cited in this background.

(1) An "abstract graph" G is a diagram consisting of a finite non-empty set of elements called, "vertices" denoted by $V(G)$ together with a set of unordered pairs of these elements, called "edges" denoted by $E(G)$. The set of vertices of the graph G is called "the vertex-set of G " and the list of the edges is called " the edge-list of G " [6,7].

(2) A "simple graph" is a graph with no loops or multiple edges [6,7]

(3) A "simple Graph on Sphere $G = (V(G) , E(G))$ consists of two finite sets $V(G)$, the vertex set of the graph , often denoted by just V , which is a nonempty set of elements called vertices, and $E(G)$, the edge set of the graph, often denoted by just E, which is a possibly empty set of elements called edges, Such that each edge e in E is completely defined by three vertices [8].

(4) A "null graph " is a graph containing no edges [6,7].

(5) A "connected graph" is a graph in one piece, whereas one which splits into several pieces is "disconnected" [6,7].

(6) Let u and w be two vertices of a graph. If u and w are joined by an edge e, then u and w are called " adjacent ".Also, u and w are called " incident with e" , and e is said to be " incident with u and w" [6,7].

(7) Let G be a graph without loops, with n-vertices labeled 1,2,3,...,n. The " adjacency matrix " $A(G)$ is defied to be the nxn matrix in which the entry in row i and column j is the number of edges joining the vertices i and j [6,7].

(8) Let G be a graph without loops, with n-vertices labeled $1,2, 3,\ldots n$ and m-edges labeled $1,2,3,...,m$. The " incidence matrix " $I(G)$ is the $m \times n$ matrix in which the entry in row i and column j is **1** if vertex i is incident with edge j and **0** otherwise [7].

(9) A "chaotic graph" is a geometric graph that carries many physical characters. [3,9,10]

(10) The " union " G of two graphs $G1$ and $G2$ is a graph $G = G1 \cup G2$ having $V(G)=V(G_1) \cup V(G_2)$ and $E(G)=E(G_1) \cup E(G_2)$ such that $V(G_1) \cap V(G_2)=\phi$ [11].

(11) The " intersection " of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, with at least one vertex in common than their intersection will be the graph of vertices and edges defined as follows:

$$V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$$

and

$$E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$$ **[12]**

3 The Main Results

In the following, we will define the chaotic graph on the sphere as the graph, which carries many physical characters. The representation of these chaotic graphs using matrices are defined. Also we will introduce some operations on chaotic graphs on a sphere such as the union and intersection as follows:

3.1 Definition

The geometric graph on a sphere ***G*** that is consisting of a finite non empty set of the elements of " point " shape called " vertices " dented by ***V(G)***together with elements of " arc " shape called " edges " denoted by ***E(G).*** When that geometric graph on a sphere carrying physical characters we call it **"a chaotic graph on sphere "**denoted by G_h. We have two types of chaotic graphs on sphere:

First type: **Internal (external) chaotic**

It is chaotic of one side (either internal or external): A physical characters carries on one side of the geometric graph on sphere like painting, printing, glue… etc.

Second type: **Two sided chaotic**

Chaotic of two sides: A physical characters carrying on two sides of a geometric graph on sphere like electricity, magnetic field…etc.

3.1.1 The matrix representation of geometric graph on sphere

Let us first give an example:

Example

Suppose that we have a graph on sphere ***G*** (simple graph) such that**:**

$V=\{v^o\ , x\ , v^1\}$ and $E=\{(v^o\, v^1)_x\ ,\ (x\, v^o)v^1\ ,\ (v^1\, x)v^o\}$. **See Fig. 1.**

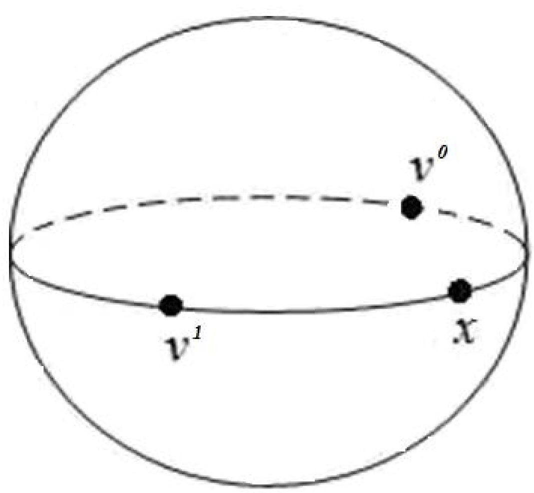


Fig. 1. Simple graph on sphere

It's adjacent and incidence matrix are:

$$A(G) = \begin{pmatrix} 0 & 1_{v^1} & 1_x \\ 1_{v^1} & 0 & 1_{v^0} \\ 1_x & 1_{v^0} & 0 \end{pmatrix}, \qquad I(G) = \begin{pmatrix} 1_x & 1_{v^1} & 0 \\ 0 & 1_{v^1} & 1_{v^0} \\ 1_x & 0 & 1_{v^0} \end{pmatrix}$$

3.1.2 The matrix representation of a chaotic graph on a sphere

Consider the chaotic graph $G_h = (V_h, E_h)$ where

$$V_h = \{ v^0_{0h}, x_{0h}, v^1_{0h} \}$$

$$, E_h = \{ (v^0_{0h}\, v^1_{0h})x_{0h}, (x_{0h} v^0_{0h})v^1_{0h}, (v^1_{0h}\, x_{0h})v^0_{0h} \},$$

This graph consists of the geometric edges

and chaotic edges

$$\{ (e^1_{0h}, e^2_{0h}, e^3_{0h}), (e^1_{1h}, e^2_{1h}, e^3_{1h}), (e^1_{2h}, e^2_{2h}, e^3_{2h}), \ldots \}$$

or $$\{ (e^1_{ih}, e^2_{ih}, e^3_{ih}), \ldots, (e^1_{ih}, e^2_{ih}, e^3_{ih}), \ldots \}$$

such that $i \in N$

See Fig. 2

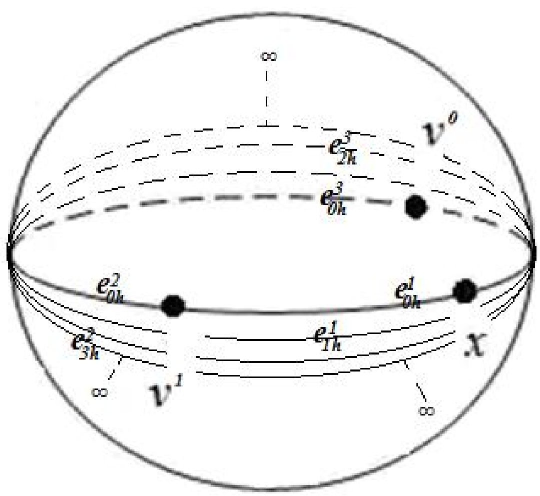


Fig. 2. Chaotic graph on sphere

The adjacent and incidence matrices of this chaotic graph on the sphere can be represented as the following:

$$A(G_h) = \begin{pmatrix} 0_{(0123\ldots\infty)h} & 1_{v^1(0123\ldots\infty)h} & 1_{x(0123\ldots\infty)h} \\ 1_{v^1(0123\ldots\infty)h} & 0_{(0123\ldots\infty)h} & 1_{v^0(0123\ldots\infty)h} \\ 1_{x(0123\ldots\infty)h} & 1_{v^0(0123\ldots\infty)h} & 0_{(0123\ldots\infty)h} \end{pmatrix}$$

$$I(G_h) = \begin{pmatrix} 1_{x(0123\ldots\infty)h} & 1_{v^1(0123\ldots\infty)h} & 0_{(0123\ldots\infty)h} \\ 0_{(0123\ldots\infty)h} & 1_{v^1(0123\ldots\infty)h} & 1_{v^0(0123\ldots\infty)h} \\ 1_{x(0123\ldots\infty)h} & 0_{(0123\ldots\infty)h} & 1_{v^0(0123\ldots\infty)h} \end{pmatrix}$$

3.2 Types of Chaotic Graphs

3.2.1 A chaotic graph has two different cases

1. All edges have the same physical character such that all $e^i_{0h}, e^i_{1h}, e^i_{2h}, \ldots, e^i_{\infty h} \; ; \; i = 1,2,3,\ldots$ have the same physical character.
2. Edges have different physical characters for example such

   e^i_{1h} represents colors, while e^i_{2h} represent electricity,… and so on.

We will discuss the first case of a chaotic graph.

3.3 Chaotic Loops on a Sphere

A loop is an edge joining a vertex to itself as shown in Fig. 3, we can represent chaotic loops on sphere as shown in Fig. 4.

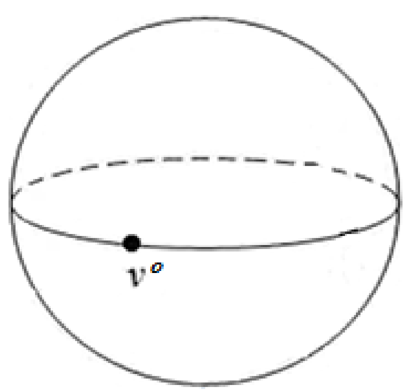


Fig. 3. Loop on sphere

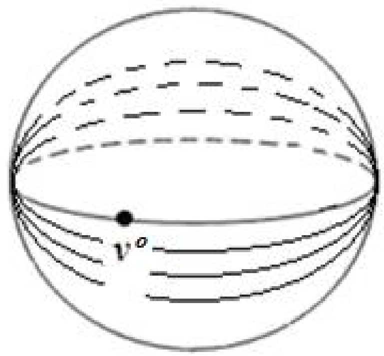


Fig. 4. Chaotic loop graph on sphere

Both incidence and adjacency matrices of the loop graph on a sphere are the same in the form:

$$I(G) = \begin{bmatrix} 1 \end{bmatrix} \qquad A(G) = \begin{bmatrix} 1 \end{bmatrix}$$

for the chaotic incidence and adjacency matrices of the chaotic loop on a sphere, are also the same form

i . e:

$$I(G_h) = \begin{bmatrix} 1_{(0\,1\,2\,3\,\ldots\,\infty)h} \end{bmatrix} \quad , \quad A(G_h) = \begin{bmatrix} 1_{(0\,1\,2\,3\,\ldots\,\infty)h} \end{bmatrix}$$

These matrices represent the chaotic of one face.

To represent a chaotic loop of two faces and its adjacent and incidence matrices for a general graph on a sphere are:

$$A(G_h) = \begin{pmatrix} 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^1(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{x(0123\ldots\infty)h} \\ 1^{(0123\ldots\infty)h}_{v^1(0123\ldots\infty)h} & 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^0(0123\ldots\infty)h} \\ 1^{(0123\ldots\infty)h}_{x(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^0(0123\ldots\infty)h} & 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} \end{pmatrix}$$

$$I(G_h) = \begin{pmatrix} 1^{(0123\ldots\infty)h}_{x(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^1(0123\ldots\infty)h} & 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} \\ 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^1(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^0(0123\ldots\infty)h} \\ 1^{(0123\ldots\infty)h}_{x(0123\ldots\infty)h} & 0^{(0123\ldots\infty)h}_{(0123\ldots\infty)h} & 1^{(0123\ldots\infty)h}_{v^0(0123\ldots\infty)h} \end{pmatrix}$$

See Fig. 5

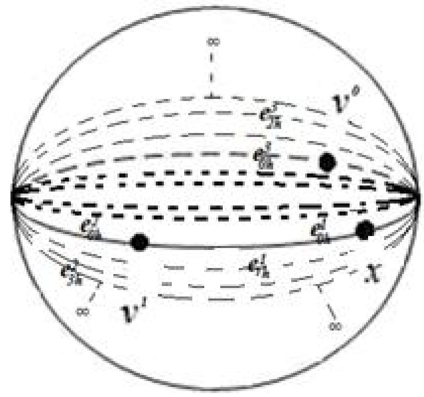


Fig. 5. Chaotic graph on sphere of two faces

The loop of a chaotic graph on a sphere is represented by the incidence and adjacent matrices as:

$$I(G_h) = \left[1^{(0\,1\,2\,3\,\ldots\,\infty)\,h}_{(0\,1\,2\,3\,\ldots\,\infty)\,h} \right] \quad , \quad A(G_h) = \left[1^{(0\,1\,2\,3\,\ldots\,\infty)\,h}_{(0\,1\,2\,3\,\ldots\,\infty)\,h} \right]$$

See Fig. 6

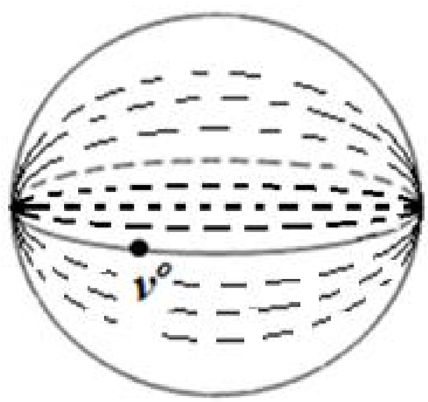


Fig. 6. Chaotic loop graph on sphere of two faces

3.4 Some Operations on Chaotic Graphs on Spheres

We will introduce some operations on chaotic graphs on a sphere which is the union of these graphs and the intersection as follows:

3.4.1 Union of chaotic graphs on the sphere

The union of two chaotic graphs on a sphere can be discussed in many cases. I will discuss some of these cases.

Case (1)

If no vertices of G^1 and G^2 in commn. See Fig. 7.

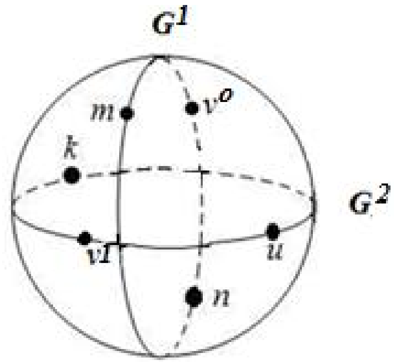


Fig. 7. Two simple graphs on sphere no vertices in common

Consider two simple graphs

$G^1 = (V^1, E^1)$ and $G^2 = (V^2, E^2)$

of two vertex-sets

$V^1 = \{v^o, n, m\}$, $V^2 = \{u, v^1, k\}$,

And two edge-set

$E^1 = \{(v^o m)_n, (n\, v^o)_m, (m\, n)_{v^o}\}$, $E^2 = \{(u\, k)v^1, (v^1 u)_k, (k\, v^1)_u\}$

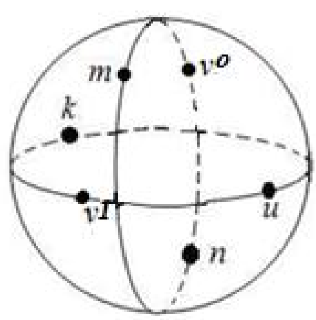
Their union is a graph of vertex-set

$V(G^1 \cup G^2) = V(G^1) \cup V(G^2) = \{v^o, n, m, u, v^1, k\}$

And an edge-set

$$E(G^1 \cup G^2) = E(G^1) \cup E(G^2)$$
$$= \{(v^o m)_n, (n\, v^o)_m, (m\, n)_{v^o}, (u\, k)_{v^1}, (v^1 u)_k, (k\, v^1)_u\}$$
$$= \{e^1, e^2, e^3, e^4, e^5, e^6\}$$

See Fig. 8



$G^1 \cup G^2$

Fig. 8. Union of two graphs on sphere G^1 and G^2 is a graph on sphere

Both incidence and adjacency matrices ***I(G*** ^{1}***)*** **,** ***A(G*** ^{1}***)*** **,** ***I(G*** ^{2}***)*** and ***A(G*** ^{2}***)*** representing the graphs ***G*** ^{1} and ***G*** ^{2} respectively, take the forms:

$$I(G^1)=\begin{array}{c} \\ v^o \\ n \\ m \end{array}\begin{array}{c} \begin{array}{ccc} e^1 & e^2 & e^3 \end{array} \\ \begin{pmatrix} 1_n & 1_m & 0 \\ 0 & 1_m & 1_{v^o} \\ 1_n & 0 & 1_{v^o} \end{pmatrix} \end{array}, \qquad A(G^1)=\begin{array}{c} \\ v^o \\ n \\ m \end{array}\begin{array}{c} \begin{array}{ccc} v^o & n & m \end{array} \\ \begin{pmatrix} 0 & 1_m & 1_n \\ 1_m & 0 & 1_{v^o} \\ 1_n & 1_{v^o} & 0 \end{pmatrix} \end{array}$$

$$I(G^2)=\begin{array}{c} \\ u \\ v^1 \\ k \end{array}\begin{array}{c} \begin{array}{ccc} e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1_{v^1} & 1_k & 0 \\ 0 & 1_k & 1_u \\ 1_{v^1} & 0 & 1_u \end{pmatrix} \end{array}, \qquad A(G^2)=\begin{array}{c} \\ u \\ v^1 \\ k \end{array}\begin{array}{c} \begin{array}{ccc} u & v^1 & k \end{array} \\ \begin{pmatrix} 0 & 1_k & 1_{v^1} \\ 1_k & 0 & 1_u \\ 1_{v^1} & 1_u & 0 \end{pmatrix} \end{array}.$$

Moreover, both incidence and adjacency matrices of the union $G^1 \cup G^2$ shown in Fig. 8 are on the following form:

$$I(G^1\cup G^2)=\begin{array}{c} \\ v^o \\ n \\ m \\ u \\ v^1 \\ k \end{array}\begin{array}{c} \begin{array}{cccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1_n & 1_m & 0 & 0 & 0 & 0 \\ 0 & 1_m & 1_{v^o} & 0 & 0 & 0 \\ 1_n & 0 & 1_{v^o} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_{v^1} & 1_k & 0 \\ 0 & 0 & 0 & 0 & 1_k & 1_u \\ 0 & 0 & 0 & 1_{v^1} & 0 & 1_u \end{pmatrix} \end{array}$$

$$A(G^1\cup G^2)=\begin{array}{c} \\ v^o \\ n \\ m \\ u \\ v^1 \\ k \end{array}\begin{array}{c} \begin{array}{cccccc} v^o & n & m & u & v^1 & k \end{array} \\ \begin{pmatrix} 0 & 1_m & 1_n & 0 & 0 & 0 \\ 1_m & 0 & 1_{v^o} & 0 & 0 & 0 \\ 1_n & 1_{v^o} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1_k & 1_{v^1} \\ 0 & 0 & 0 & 1_k & 0 & 1_u \\ 0 & 0 & 0 & 1_{v^1} & 1_u & 0 \end{pmatrix} \end{array}$$

Then we arrive to the following theorem:

Theorem 1:

If ***A(*** G_1 ***)*** and ***A(*** G_2 ***)*** are the adjacency matrices for the two graphs G_1 and G_2, respectively on a sphere, and if ***I(*** G_1 ***)*** and ***(*** G_2 ***)*** are the incidence matrices of the two graphs G_1 and G_2, respectively then:

The adjacency and incidence matrices of $G_1 \cup G_2$ are:

$$\begin{pmatrix} A(G_1) & 0 \\ 0 & A(G_2) \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} I(G_1) & 0 \\ 0 & I(G_2) \end{pmatrix}$$

As previously mentioned "the chaotic graph" is a geometric graph that carries many characters [13], then the chaotic graphs corresponding the two graphs G^1 and G^2 respectively, are:

$$G_h^1 = (V_h^1, E_h^1) \quad \text{and} \quad G_h^2 = (V_h^2, E_h^2)$$

Such that

$$V_h^1 = \{ v_{jh}^o, n_{jh}, m_{jh} : j = 0,1,2,\ldots \}, \quad E_h^1 = \{ e_{jh}^1, e_{jh}^2, e_{jh}^3 : j = 0,1,2,\ldots \}$$

$$, V_h^2 = \{ u_{jh}, v_{jh}^1, k_{jh} : j = 0,1,2,\ldots \} \text{ and } E_h^2 = \{ e_{jh}^4, e_{jh}^5, e_{jh}^6 : j = 0,1,2,\ldots \}$$

It is obvious v_{jh}^i is the chaotic image of the vertex v_{oh}^i, where each vertex for v_{jh}^i example is overlapped v^i on $; i, j = 0,1,2,\ldots$ See Fig. 9.

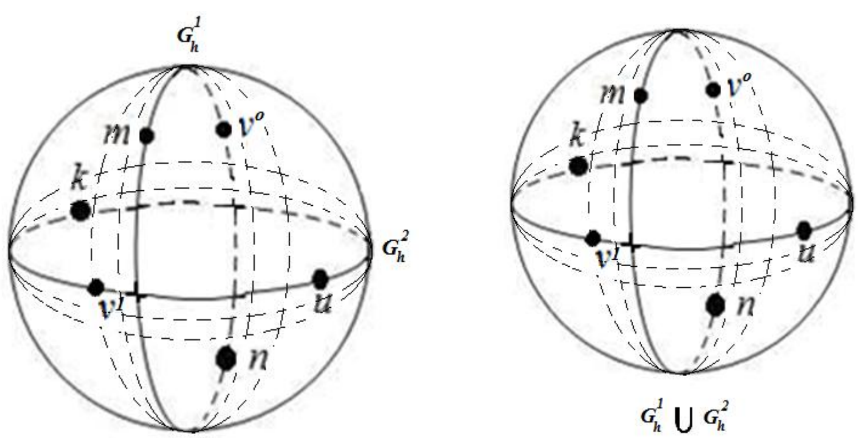


Fig. 9. Chaotic graphs corresponding to Figs. 7 and 8

Both of the chaotic incidence and adjacency matrices of the chaotic graphs G_h^1, G_h^2 and $G_h^1 \cup G_h^2$ are :

$$I_{1h} = \begin{pmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{pmatrix}, \quad A_{1h} = \begin{pmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{pmatrix}$$

$$I_{2h} = \begin{pmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{pmatrix}, \quad A_{2h} = \begin{pmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{pmatrix}$$

$$I_h = \begin{pmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{pmatrix}$$

$$A_h = \begin{pmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{pmatrix}$$

Noting that the symbol $1_{012\ldots\infty h}$ denotes that we have an infinite number of chaotic Graphs overlapped on the geometric graph. Also, note that the symbol $0_{012\ldots\infty h}$ is equivalent to the symbol $\mathbf{0}$ which means that we do not have an edge incident with the corresponding vertices.

Then we have a similar result for the chaotic graphs:

Theorem 2:

The chaotic matrices of both incidence and adjacency representing the union of two chaotic graphs can be obtained by the chaotic incidence and adjacency matrices representing each of the given chaotic graphs forming such union.

i.e If $A_h(G^1)$ and $A_h(G^2)$ are the chaotic adjacency matrices for the two chaotic graphs G_h^1 and G_h^2, respectively on a sphere. And if $I_h(G^1)$ and $I_h(G^2)$ are the incidence matrices of the two graphs G_h^1 and G_h^2, respectively then:

The chaotic adjacency and chaotic incidence matrices of $G_h^1 \cup G_h^2$ are:

$$\left(\begin{array}{c|c} A_h(G^1) & 0_h \\ \hline 0_h & A_h(G^2) \end{array}\right) \quad \text{and} \quad \left(\begin{array}{c|c} I_h(G^1) & 0_h \\ \hline 0_h & I_h(G^2) \end{array}\right)$$

<u>**Case (2)**</u>

If the two graphs G^1 and G^2 have a common vertex. See Fig. 10.

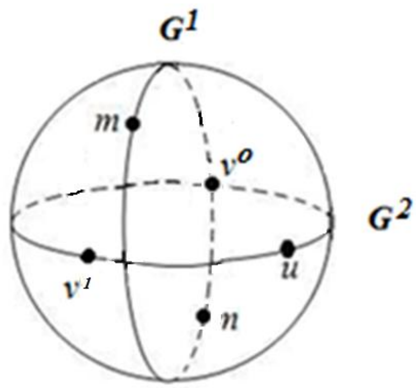


Fig. 10. Two simple graphs on sphere are common in a one vertex

Consider two simple graphs $G^1 = (V^1, E^1)$ and $G^2 = (V^2, E^2)$ with a vertex in common,

where:

$V(G^1) = \{n, m, v^o\}$,
$V(G^2) = \{v^o, u, v^1\}$,

$E(G^1) = \{e^1 = (n\, v^o)_m,\ e^2 = (m\, n)_{v^o},\ e^3 = (v^o\, m)_n\}$ and
$E(G^2) = \{e^4 = (v^o v^1)_u,\ e^5 = (u\, v^o)_{v^1},\ e^6 = (v^1\, u)_{v^o}\}$

The union is the graph of a vertex-set

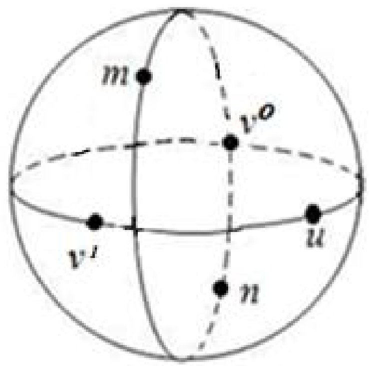
$V(G^1 \cup G^2) = V(G^1) \cup V(G^2) = \{n, m, v^o, u, v^1\}$, and an edge-set

$E(G^1 \cup G^2) = E(G^1) \cup E(G^2)$

$= \{(n\, v^o)_m, (m\, n)_{v^o}, (v^o\, m)_n, (v^o v^1)u, (u\, v^o)_{v^1}, (v^1\, u)_{v^o}\}$

$= \{e^1, e^2, e^3, e^4, e^5, e^6\}$

See Fig. 11



$G^1 \cup G^2$

Fig. 11. Union of two graphs on sphere G^1 and G^2 is a graph on sphere

Both incidence and adjacency matrices $I(G^1)$, $A(G^1)$, $I(G^2)$ and $A(G^2)$ are representing the graphs G^1 and G^2 respectively, take the forms:

$$I(G^1) = \begin{array}{c} \\ n \\ m \\ v^o \end{array} \begin{array}{c} \begin{matrix} e^1 & e^2 & e^3 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \end{array}, \qquad A(G^1) = \begin{array}{c} \\ n \\ m \\ v^o \end{array} \begin{array}{c} \begin{matrix} n & m & v^o \end{matrix} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$I(G^2) = \begin{array}{c} \\ v^o \\ u \\ v^1 \end{array} \begin{array}{c} \begin{matrix} e^4 & e^5 & e^6 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \end{array}, \qquad A(G^2) = \begin{array}{c} \\ v^o \\ u \\ v^1 \end{array} \begin{array}{c} \begin{matrix} v^o & u & v^1 \end{matrix} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

Moreover, both incidence and adjacency matrices of the union $G^1 \cup G^2$ in the following forms:

$$I(G^1 \cup G^2) = \begin{array}{c} \\ n \\ m \\ v^o \\ u \\ v^1 \end{array} \begin{array}{c} \begin{array}{cccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

$$A(G^1 \cup G^2) = \begin{array}{c} \\ n \\ m \\ v^o \\ u \\ v^1 \end{array} \begin{array}{c} \begin{array}{ccccc} n & m & v^o & u & v^1 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Note that we get one entry in common in the matrix $A(G^1 \cup G^2)$ since we have a common vertex in G^1 and G^2.

Then we arrive to the following theorem:

Theorem 3:

The matrices of both incidence and adjacency that are representing the union of two simple graphs on a sphere with a common vertex can be obtained by the incidence and adjacency matrices that are representing each of the given graphs. The induced graph consists of an entry shown the common vertex as illustrated in $A(G^1 \cup G^2)$

Now we will discuss the union of chaotic graphs.

Let G_h^1 , G_h^2 be two chaotic graphs See Fig. 12.

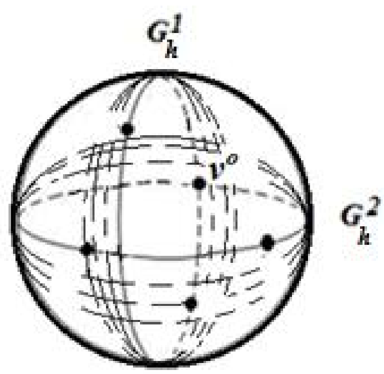
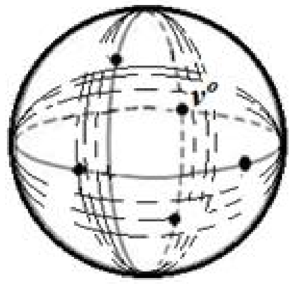


Fig. 12. Chaotic graphs corresponding to Fig. 10

The chaotic union $G^1 \cup G^2$ is illustrated in Fig. 13.



$G_h^1 \cup G_h^2$

Fig. 13. Chaotic graphs corresponding to Fig. 11

Both of the chaotic incidence and adjacency matrices of the chaotic graphs G_h^1 and G_h^2 are:

$$I_h^1 = \begin{pmatrix} 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \end{pmatrix}, \qquad A_h^1 = \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix}$$

$$I_h^2 = \begin{pmatrix} 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \end{pmatrix}, \qquad A_h^2 = \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix}$$

where, $\boldsymbol{1_h = 1_{0123\ldots\infty}}$ and $\boldsymbol{0_h = 0_{0123\ldots\infty}}$.

On the other hand, both chaotic incidence and adjacency matrices of the chaotic union $G_h^1 \cup G_h^2$ has of the following forms:

$$I_h = \begin{array}{c} \\ n \\ m \\ v^o \\ u \\ v^1 \end{array} \begin{pmatrix} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 \\ 1_h & 1_h & 0_h & 0_h & 0_h & 0_h \\ 0_h & 1_h & 1_h & 0_h & 0_h & 0_h \\ 1_h & 0_h & 1_h & 1_h & 1_h & 0_h \\ 0_h & 0_h & 0_h & 0_h & 1_h & 1_h \\ 0_h & 0_h & 0_h & 1_h & 0_h & 1_h \end{pmatrix}$$

$$A_h = \begin{array}{c} \\ n \\ m \\ v^o \\ u \\ v^1 \end{array} \begin{pmatrix} n & m & v^o & u & v^1 \\ 0_h & 1_h & 1_h & 0_h & 0_h \\ 1_h & 0_h & 1_h & 0_h & 0_h \\ 1_h & 1_h & 0_h & 1_h & 1_h \\ 0_h & 0_h & 1_h & 0_h & 1_h \\ 0_h & 0_h & 1_h & 1_h & 0_h \end{pmatrix}$$

From the above discussion, we have the following result for the chaotic graphs:

Theorem 4:

The chaotic matrices of both incidence and adjacency representing the union of some chaotic graphs can be obtained by the chaotic incidence and adjacency matrices that are representing each of the given chaotic graphs forming such union.

We observe from Figs. 12 and 13 that we have two cases:

(i) The two chaotic graphs G_h^1 and G_h^2 are of the same system of physical characters, then the chaotic graph $G_h^1 \cup G_h^2$ has one system of physical characters; we can call it **"homogeneous"**.

(ii) The two chaotic graphs G_h^1 and G_h^2 are of two different systems of physical characters, then the chaotic graph $G_h^1 \cup G_h^2$ is of two different systems of physical characters, we can call it **"nonhomogeneous"**.

Case (3)

If the two graphs G^1 and G^2 have two vertices in common. See Fig. 14.

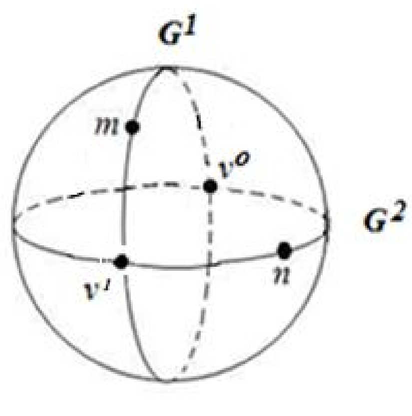


Fig. 14. Two simple graphs on a sphere with two vertices in common

Consider another two simple graphs on the sphere, where $G^1 = (V^1, E^1)$ and $G^2 = (V^2, E^2)$, such that:

$$V(G^1) = \{v^o, v^1, m\} \;,$$
$$V(G^2) = \{v^o, n, v^1\} \;,$$

$$E(G1) = \{(v^o m)_v{}^1, (v^1 v^o)_m, (m\, v^1)_v{}^o\} = \{e^1, e^2, e^3\}$$

and

$$E(G^2) = \{(v^o v^1)_n, (n\, v^o)_v{}^1, (v^1 n)_v{}^o\} = \{e^4, e^5, e^6\}$$

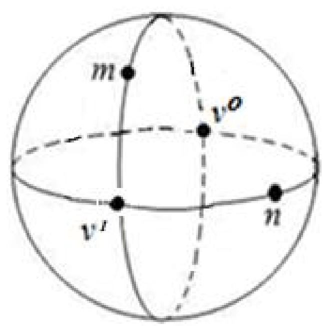
Their union is a graph of a vertex-set

$$V(G^1 \cup G^2) = V(G^1) \cup V(G^2) = \{m, v^o, v^1, n\} \;,$$

and an edge-set

$$E(G^1 \cup G^2) = E(G^1) \cup E(G^2) = \{(v^o m)_v{}^1, (v^1 v^o)_m, (m\, v^1)_v{}^o, (v^o v^1)_n, (n\, v^o)_v{}^1, (v^1 n)_v{}^o\} = \{e^1, e^2, e^3, e^4, e^5, e^6\} \;.$$

See Fig. 15



$G^1 \cup G^2$

Fig. 15. Union of two graphs on sphere G^1 and G^2 with two vertices in common

Both the incidence and adjacency matrices representing the above three graphs G^1, G^2 and $G^1 \cup G^2$ in Figs. 14 and 15 take the following forms:

$$I(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array} \begin{array}{c} \begin{matrix} e^1 & e^2 & e^3 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array} \;, \qquad A(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array} \begin{array}{c} \begin{matrix} m & v^o & v^1 \end{matrix} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$I(G^2) = \begin{array}{c} \\ v^1 \\ v^o \\ n \end{array} \begin{array}{c} \begin{array}{ccc} e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array} , \qquad A(G^2) = \begin{array}{c} \\ v^o \\ v^1 \\ n \end{array} \begin{array}{c} \begin{array}{ccc} v^o & v^1 & n \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$I(G^1 \cup G^2) = \begin{array}{c} \\ m \\ v^o \\ v^1 \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

$$A(G^1 \cup G^2) = \begin{array}{c} \\ m \\ v^o \\ v^1 \\ n \end{array} \begin{array}{c} \begin{array}{cccc} m & v^o & v^1 & n \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

We notice that the adjacency matrix of the union $G^1 \cup G^2$ has a square matrix of size **2X2.** Moreover, the chaotic graphs corresponding the three graphs G^1, G^2 and $G^1 \cup G^2$ shown in Figs. 14 and 15 are G^1_h, G^2_h and $G^1_h \cup G^2_h$

See Fig. 16

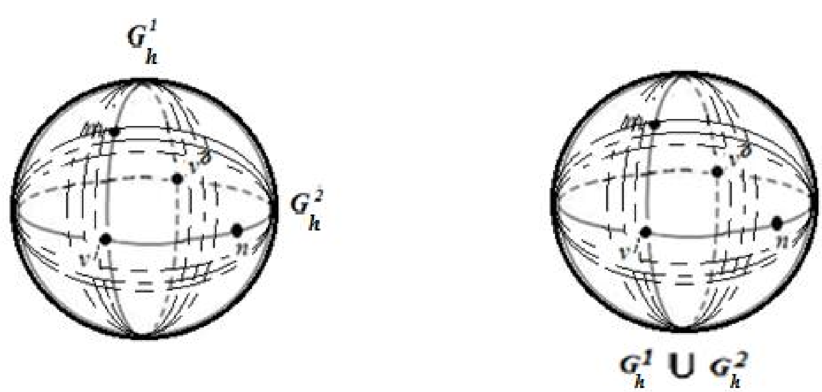


Fig. 16. Chaotic graphs corresponding to Figs. 14 and 15

The chaotic incidence and adjacency matrices of the chaotic graphs G^1_h, G^2_h and $G^1_h \cup G^2_h$ are:

$$I_h(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array}\begin{array}{c} \begin{array}{ccc} e^1 & e^2 & e^3 \end{array} \\ \begin{pmatrix} 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \end{pmatrix} \end{array}, \quad A_h(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array}\begin{array}{c} \begin{array}{ccc} m & v^o & v^1 \end{array} \\ \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix} \end{array}$$

$$I_h(G^2) = \begin{array}{c} \\ v^1 \\ v^o \\ n \end{array}\begin{array}{c} \begin{array}{ccc} e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \end{pmatrix} \end{array}, \quad A_h(G^2) = \begin{array}{c} \\ v^o \\ v^1 \\ n \end{array}\begin{array}{c} \begin{array}{ccc} v^o & v^1 & n \end{array} \\ \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix} \end{array}$$

$$I_h(G^1 \cup G^2) = \begin{array}{c} \\ m \\ v^o \\ v^1 \\ n \end{array}\begin{array}{c} \begin{array}{cccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1_h & 0_h & 1_h & 0_h & 0_h & 0_h \\ 1_h & 1_h & 0_h & 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h & 1_h & 0_h & 1_h \\ 0_h & 0_h & 0_h & 0_h & 1_h & 1_h \end{pmatrix} \end{array}$$

$$A_h(G^1 \cup G^2) = \begin{array}{c} \\ m \\ v^o \\ v^1 \\ n \end{array}\begin{array}{c} \begin{array}{cccc} m & v^o & v^1 & n \end{array} \\ \begin{pmatrix} 0_h & 1_h & 1_h & 0_h \\ 1_h & 0_h & 1_h & 1_h \\ 1_h & 1_h & 0_h & 1_h \\ 0_h & 1_h & 1_h & 0_h \end{pmatrix} \end{array}$$

3.4.2 Intersection of chaotic graphs on spheres

The study the intersection of two chaotic graphs can be split into the following cases:

Case (1):

The chaotic graphs are disjoint

Suppose that we have two graphs

$G^1 = (V^1, E^1)$ and $G^2 = (V^2, E^2)$

of two vertex-sets

$V^1 = \{v^o, n, m\}$, $V^2 = \{u, v^1, k\}$, and two edge-sets

$$\boldsymbol{E}^{1} = \{ (v^{o}\, m)_{n} ,\ (n\, v^{o})_{m} ,\ (m\, n)_{v^{o}} \} = \{ e_{1} , e_{2} , e_{3} \} ,$$

$$\boldsymbol{E}^{2} = \{ (u\, k)v^{1} ,\ (v^{1}\, u)_{k} ,\ (k\, v^{1})_{u} \} = \{ e_{3}, e_{4}, e_{5} \}$$

It is clear that the intersection $\boldsymbol{G}^{1} \cap \boldsymbol{G}^{2} = \phi$, is the empty graph see Fig. 17.

Fig. 17. Intersection of disjoint graphs

Both the incidence and adjacency matrices representing these graphs are:

$$I_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} , \qquad A_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$I_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} , \qquad A_2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

The matrices of the intersection will be called **ϕ**-matrices, i.e. ***I* = ϕ**-*matrix* and ***A* = ϕ**-*matrix*

We mean by **ϕ**-*matrix* that we have neither vertices nor edges.

The chaotic graphs corresponding to the above two geometric graphs $\boldsymbol{G}^{1}$ and $\boldsymbol{G}^{2}$ in Fig. 17 are G_h^1 and G_h^2 respectively, such that

$$V_h^1 = \{ v_{jh}^{o} ,\ n_{jh} ,\ m_{jh} : j = 0,1,2,\ldots \} ,\quad E_h^1 = \{ e_{jh}^{1} ,\ e_{jh}^{2} ,\ e_{jh}^{3} : j = 0,1,2,\ldots \} ,$$

$$V_h^2 = \{ u_{jh},\ v_{jh}^1,\ k_{jh}\ : j = 0,1,2,\ldots \} \quad \text{and} \quad E_h^2 = \{ e_{jh}^4,\ e_{jh}^5,\ e_{jh}^6\ : j = 0,1,2,\ldots \}$$

See Fig. 18

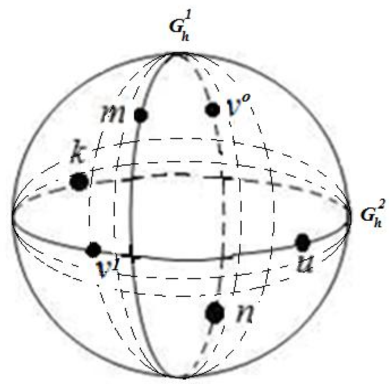


Fig. 18. Intersection of disjoint chaotic graphs

The intersection $G_h^1 \cap G_h^2$ will be the chaotic empty graph, and both the chaotic incidence and adjacency matrices represented the chaotic graphs and G_h^1, G_h^2 and $G_h^1 \cap G_h^2$ are:

$$I_{1h} = \begin{pmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{pmatrix}, \quad A_{1h} = \begin{pmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{pmatrix}$$

$$I_{2h} = \begin{pmatrix} 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{pmatrix} \quad A_{2h} = \begin{pmatrix} 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{pmatrix}.$$

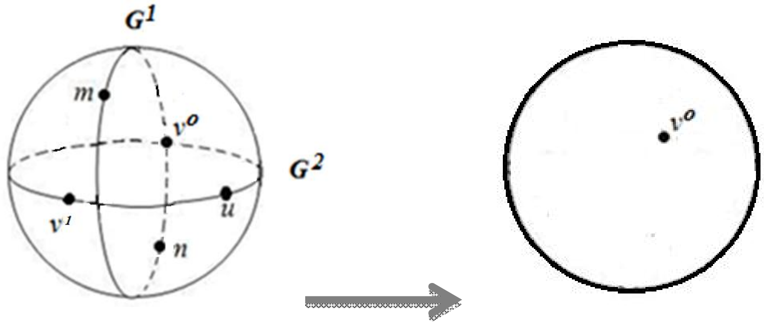
It is also clear that the chaotic matrices of the intersection will be **ϕ**-*matrices*,

i.e. I **= ϕ-*matrix*** and A **= ϕ-*matrix.***

<u>**Case (2):**</u>

The chaotic graphs with a common vertex

Let the graph G in Fig. 19 represent the two graphs G^1 and G^2.



$G_h^1 \cap G_h^2$

Fig. 19. Intersection of two graphs in a vertex

The intersection $G^1 \cap G^2$ is obviously the a null graph { v^o }. Both the incidence and adjacency matrices of this graph are:

$$I(G^1)=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad A(G^1)=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$I(G^2)=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad A(G^2)=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

and $I = [\ 0\ ]$, $A = [\ 0\ ]$.Clearly, both the incidence and adjacency matrices representing the intersection, are sub matrices of the corresponding matrices representing the given two graphs, respectively.

The chaotic graphs corresponding the two geometric graphs $G^1$and G^2 shown in Fig. 19 are G_h^1 and G_h^2 respectively. Their intersection $G_h^1 \cap G_h^2$ is the chaotic null graph N_{ih} of only one vertex v_{ih}^o which carries many physical characters. See Fig. 20.

$G_h^1 \cap G_h^2$

Fig. 20. The chaotic graphs are common in a chaotic vertex

The chaotic incidence and adjacency matrices represented the chaotic graphs G_h^1, G_h^2 and $G_h^1 \cap G_h^2$ are:

$$I_{1h} = \begin{pmatrix} 1_{(012..\infty)h} & 1_{(012..\infty)h} & 0_{(012..\infty)h} \\ 0_{(012..\infty)h} & 1_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 0_{(012..\infty)h} & 1_{(012..\infty)h} \end{pmatrix}, \quad A_{1h} = \begin{pmatrix} 0_{(012..\infty)h} & 1_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 0_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 1_{(012..\infty)h} & 0_{(012..\infty)h} \end{pmatrix}$$

$$I_{2h} = \begin{pmatrix} 1_{(012..\infty)h} & 1_{(012..\infty)h} & 0_{(012..\infty)h} \\ 0_{(012..\infty)h} & 1_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 0_{(012..\infty)h} & 1_{(012..\infty)h} \end{pmatrix}, \quad A_{2h} = \begin{pmatrix} 0_{(012..\infty)h} & 1_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 0_{(012..\infty)h} & 1_{(012..\infty)h} \\ 1_{(012..\infty)h} & 1_{(012..\infty)h} & 0_{(012..\infty)h} \end{pmatrix}$$

and

$$I_h = [\,0_{(012..\infty)h}\,], \quad A_h = [\,0_{(012..\infty)h}\,]$$

Case (3):

The chaotic graphs are intersected in two vertices

Consider the intersection of two graphs G^1 and G^2 shown in Fig. 21.

where

$$G^1 \cap G^2 = \{ v^o , v^1 \}.$$

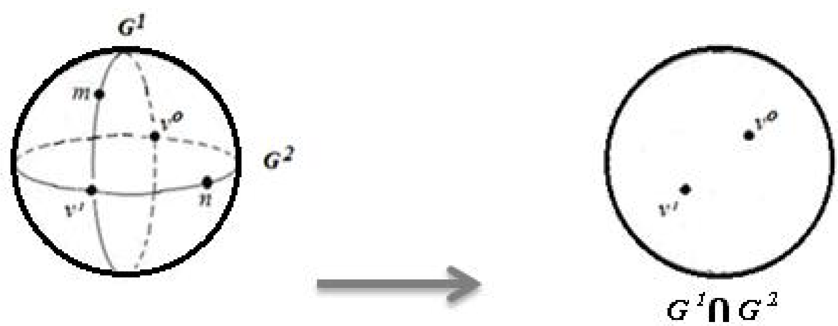


Fig. 21. Intersection of two graphs in two vertices

Both the incidence and adjacency matrices representing these graphs are:

$$I(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array} \begin{array}{c} \begin{array}{ccc} e^1 & e^2 & e^3 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array}, \quad A(G^1) = \begin{array}{c} \\ m \\ v^o \\ v^1 \end{array} \begin{array}{c} \begin{array}{ccc} m & v^o & v^1 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$I(G^2) = \begin{array}{c} \\ v^1 \\ v^o \\ n \end{array} \begin{array}{c} \begin{array}{ccc} e^4 & e^5 & e^6 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array}, \quad A(G^2) = \begin{array}{c} \\ v^o \\ v^1 \\ n \end{array} \begin{array}{c} \begin{array}{ccc} v^o & v^1 & n \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

and $\boldsymbol{I} = \begin{bmatrix} 0 & 0 \end{bmatrix}$, $\boldsymbol{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

From all the above, we formulate the following theorem:

Theorem 5:

If any two graphs are intersected in a vertex or more without any common edges, then both of the incidence and adjacency matrices representing that intersection are zero matrices.

The chaotic graphs corresponding to the above geometric graphs G^1 and G^2 in Fig. 21 are G_h^1 and G_h^2 respectively. Then the intersection $G_h^1 \cap G_h^2$ is the chaotic null graph N_{2h} of only two chaotic vertices v_{ih}^o and v_{ih}^1 for $i = 0,1,2,\ldots$ See Fig. 22.

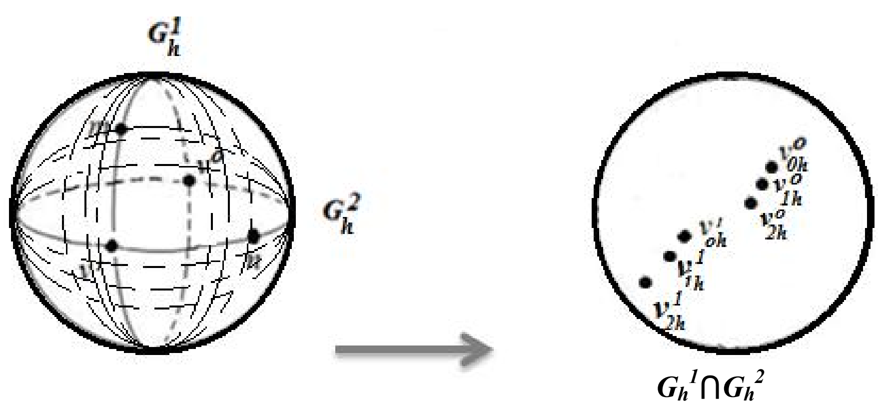


Fig. 22. The chaotic graphs are intersected in only two chaotic vertices

Also, the chaotic incidence and adjacency matrices for these three chaotic graphs G_h^1, G_h^2 and $G_h^1 \cap G_h^2$ are:

$$I_h(G^1) = \begin{pmatrix} 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \end{pmatrix}, \quad A_h(G^1) = \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix}$$

$$I_h(G^2) = \begin{pmatrix} 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \\ 0_h & 1_h & 1_h \end{pmatrix}, \quad A_h(G^2) = \begin{pmatrix} 0_h & 1_h & 1_h \\ 1_h & 0_h & 1_h \\ 1_h & 1_h & 0_h \end{pmatrix}$$

and

$$I_h = \begin{pmatrix} 0_h & 0_h \end{pmatrix}, \quad A_h = \begin{pmatrix} 0_h & 0_h \\ 0_h & 0_h \end{pmatrix}$$

Therefore, we have

Theorem 6:

If any two chaotic graphs are intersected in a common vertex or more vertices without any common chaotic edges, then both of the chaotic incidence and adjacency matrices representing that intersection are a special type of zero matrices.

4 Some Applications

(1) The bonds of chemical reactions are represented by the graph, but an infinite chaos of bonds, which represent the physical characters such that the vertices represent the components of the complex compound and the edges represent the chaotic physical characters like the radiation, the flow of flux...are represented by the chaotic graph. By the modeling of this graph by incidence and adjacency matrices, one can computerize these matrices to discuss these physical properties of the atoms. See Fig. 23.
(2) The biological properties of biological isomers represent a chain of1-chaotic graphs, which represent the biological properties like link, growth toxics, and pharmaceuticals.
(3) The orbital system of one atom represents a graph of the nucleus and electrons. There are some connections between nucleus and electron, and electron and electron that represent a complete graph. See Fig. 24.

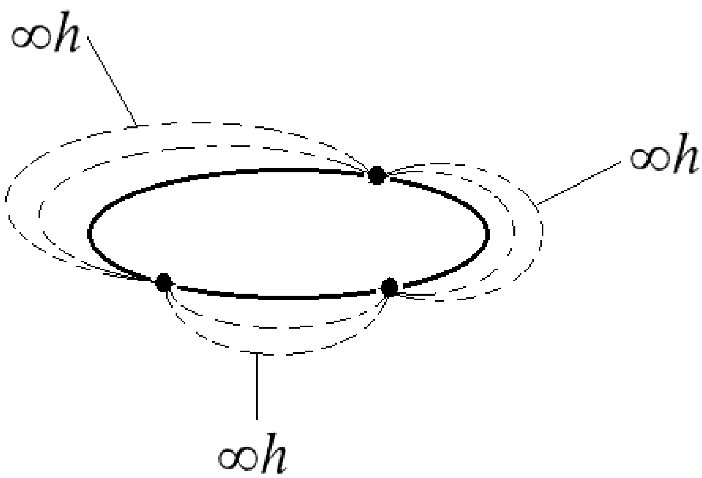


Fig. 23. Bonds in chemical reactions

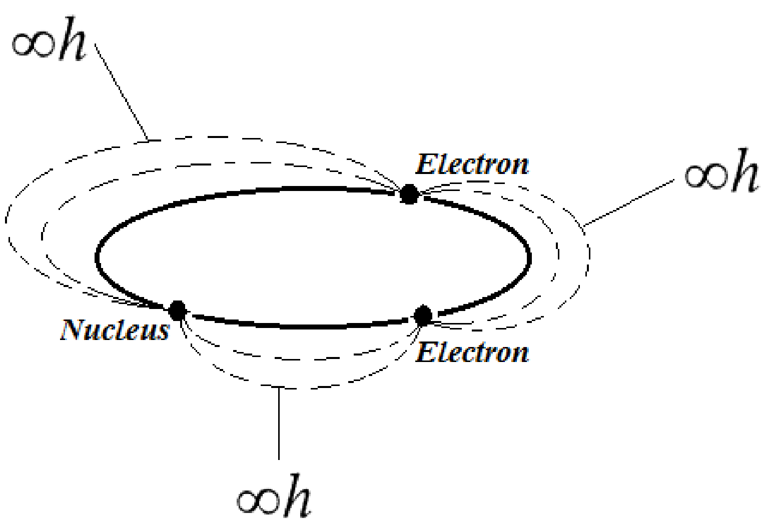


Fig. 24. The orbital system of the atom

5 Conclusion

In this paper, we studied the union of chaotic graphs on a sphere. Also, we discussed all cases of intersection of them. Moreover, many theorems governing all the probabilities are deduced and given some applications in physics, chemistry and biology.

Competing Interests

Authors have declared that no competing interests exist.

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