Journal of Advances in Mathematics and Computer Science

27(2): 1-15, 2018; Article no.JAMCS.40808

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Bi-Level Multi-Objective Large Scale Integer Quadratic Programming Problem with Symmetric Trapezoidal Fuzzy Numbers in the Objective Functions

O. E. Emam[1], E. Fathy[2] and A. A. Abdullah[3*]

[1]*Department of Information Systems, Faculty of Computer Science and Information, Helwan University, P.O.Box 11795, Egypt.*

[2]*Department of Mathematics, Faculty of Science, Helwan University, P.O.Box 11795, Egypt.*

[3]*Department of Information Systems, Higher Institute of Computer Science and Information Systems, Fifth Avenue - New Cairo, Egypt.*

Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2018/40808

Editor(s):

(1) Junjie Chen, Professor, Department of Electrical Engineering, University of Texas at Arlington, USA.

Reviewers:

(1) Piyush Shroff, Texas State University, USA.

(2) Tunjo Perić, University of Zagreb, Croatia.

Complete Peer review History: http://www.sciencedomain.org/review-history/24259

Original Research Article

Received: 31st January 2018

Accepted: 13th April 2018

Published: 21st April 2018

Abstract

This paper focuses on the solution of a Bi-Level Multi-Objective Large Scale Integer Quadratic Programming (BLMOLSIQP) problem, where all the decision parameters in the objective functions are symmetric trapezoidal fuzzy numbers, and have block angular structure of the constraints. The suggested algorithm based on α-level sets of fuzzy numbers, weighted sum method, Taylor's series, Decomposition algorithm, and also the Branch and Bound method is used to find a compromised solution for the problem under consideration. Then, the proposed algorithm is compared to Frank and Wolfe algorithm to demonstrate its effectiveness. Moreover, the theoretical results are illustrated with the help of a numerical example.

Keywords: Large scale; integer programming; quadratic programming; multi-objective; fuzzy programming; bi-level programming.

**Corresponding author: E-mail: abdallah_lovers12@yahoo.com;*

MSC 2010: 90C06, 90C10, 90C20, 90C29, 90C70, 90C99.

1 Introduction

The concept of Fuzzy linear programming problems is essential for fuzzy modeling which can formulate uncertainty in actual environment. Afterwards, many authors [1-5] considered various types of the fuzzy linear programming and proposed several approaches for solving these problems.

Bi-level mathematical programming (BLMP) is defined as mathematical programming that solves decentralized planning problems with two decision makers (DMs) in two levels or hierarchical organization. The basic concept of BLMP is that the upper-level decision maker (ULDM) - A.K.A. the leader sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the decisions of the lower-level decision maker (LLDM) - A.K.A. the follower are then submitted and modified by the ULDM with the consideration of the overall benefit of the organization; the process is continued until a satisfactory solution is reached [6,7].

Optimization problems which have a large number of variables and a large number of constraints are called Large-Scale Programming Problems (LSPP). Large-scale systems arise in real-life problems while dealing with applications such as natural resource management, manpower planning, industrial planning, control of multi period production, and inventory problems [7]. One prominent structure of the LSPP is the block angular structure. In this structure, an LSPP is separated into smaller sub-problems which appear together, sharing common resources in the upper-most interconnected constraints [8,9].

A Quadratic programming problem in which some or all of the variables must take non-negative integer values is commonly known as Integer Quadratic Programming Problem (IQPP). If all the variables are constrained to be integer; it is called a Pure Integer Programming Problem. In some situations, each variable can take on the values of either zero or one as in ‘do’ or ‘not to do’. all type decision and such problems are referred to Zero-One Programming Problem or Standard Discrete Programming Problem.

A systematic procedure for solving integer programming problem was first developed by R.E. Gomory in the year 1958 [10]. Extending the procedure to solve the Mixed-Integer Programming Problem. He also derived algorithms (named as cutting-plane algorithm). Later on, an efficient method with relatively new approach was developed known as “Branch and Bound Method” [11].

Emam presented a bi-level integer non-linear programming problem with linear or non-linear constraints [12] and proposed an interactive approach to solve a bi-level integer multi-objective fractional programming problem in [13]. Baky [14] introduced two new algorithms to solve multi-level multi-objective linear programming problems through the fuzzy goal programming approach. The membership functions for the defined fuzzy goals of all objective functions at all levels were developed. Then the fuzzy goal programming approach was used to obtain the satisfactory solution for all decision makers.

Abo–Sinna and Abou-El-Enien [15] extend TOPSIS for solving interactive large scale multiple Objective programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the α-Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the α-Level sets of fuzzy numbers. An interactive fuzzy decision making algorithm for generating α-Pareto optimal solution through TOPSIS approach is provided where the decision maker (DM) is asked to specify the degree α and the relative importance of objectives.

Osman et al. [16] presented a method for solving a special class of large scale fuzzy multi-objective integer problems depending on the decomposition algorithm.

Emam et al. [17] solved a Fully Rough Three Level Large Scale Integer Linear Programming (FRTLLSILP) problem, in which all decision parameters and decision variables in the objective functions and the constraints are rough intervals, and have block angular structure of the constraints. The optimal values of decision rough variables are rough integer intervals. The proposed model was based on interval method and slice-sum method in an interactive model to find a compromised solution for (FRTLLSILP).

Emam et al. [18] solved a Fully Fuzzy Multi-Level Linear Programming (FFMLLP) Problem, where all of its decision parameters and variables are fuzzy numbers. It is an algorithm depending on the fuzzy decision approach and bound and decomposition method to find a fuzzy optimal solution.

The rest of this paper is organized as follows: we start in Section 2 by formulating the model of (BLMOLSIQP) problem with fuzzy parameters in the objective functions. The theories used α-level sets to transform fuzzy number in the objective functions into deterministic form are obtained in section 3. Section 4 presents a Taylor series approach for (BLMOLSIQP) problem then converts the quadratic objective functions and using the weighted sum method to transform the objective functions from multi-objective form to single objective form. Section 5 presents a decomposition algorithm for a Bi-level Large Scale Integer Linear Programming (BLLSILP) problem. Section 6 involves the concepts of Frank and Wolfe algorithm. An algorithm followed by a flowchart for solving the proposed problem is suggested in Section 7 and Section 8. In addition, a numerical example is provided in Section 9 to clarify the results. Finally, a conclusion and future works are reported in Section 10.

2 Problem Formulation

Assume that, $F_i : R^m \to R$, (i=1,2) are the first level objective function and the second level objective function . $x_j \in R^n, (j = 1,2,..., m)$ be a real vector decision variables indicating the control of each level. Therefore, the first level decision maker (FLDM) has control over the vector x_1, x_2 and the second level decision maker (SLDM) has control over the vectors x_3, x_4. $\tilde{u}_{ik}$ n-dimensional row vector (i=1,2) (k=1,2,.., n) of fuzzy parameters in the objective functions. $\tilde{A}_{ik}$ are 1×m matrices of fuzzy parameters coefficients (i=1,2) (k=1,2,.., n). $\tilde{L}_{ik}$ are m×m real matrices describing the fuzzy coefficient of the quadratic terms (i=1,2) (k=1,2,.., n). G is the large scale linear constraint set where, $b = (b_0,...,b_m)^T$ is $(m+1)$ vector, and $a_{01},...,a_{0m}, d_1,...,d_m$ are constants .Therefore, the Bi-Level Multi-Objective Large Scale Integer Quadratic Programming (BLMOLSIQP) problem with symmetric trapezoidal fuzzy numbers in the objective functions may be formulated as follows:

[FLDM]

$$\underset{x_1,x_2}{Max} \; F_1(x,\tilde{u}) = \underset{x_1,x_2}{Max} \, [f_{1k}(x_j,\tilde{u}_{1k}) = \tilde{A}(\tilde{u}_{1k}) \; x + \frac{1}{2} x^T \tilde{L}_{1k} x] \tag{1}$$

Where $x_3,...,x_m$ solves

[SLDM]

$$\underset{x_3,x_4}{Max} \; F_2(x,\tilde{u}) = \underset{x_3,x_4}{Max} \, [f_{2k}(x_j,\tilde{u}_{2k}) = \tilde{A}(\tilde{u}_{2k}) \; x + \frac{1}{2} x^T \tilde{L}_{2k} x] \tag{2}$$

Where $x_5,...,x_m$ solves

Subject to

$$x \in G = \{ a_{01}x_1 + a_{02}x_2 \qquad + a_{0m}x_m \leq b_0,$$
$$d_1x_1 \qquad \leq b_1,$$
$$d_2x_2 \qquad \leq b_2,$$
$$\vdots$$
$$d_m x_m \leq b_m,$$
$$x_1,\dots,x_m \geq 0 \text{ and integer}\}. \tag{3}$$

Definition 1. [13]

Let G_1, G_2 be the feasible regions of FLDM and SLDM, for any $\left(x_1, x_2 \in G_1 = \{x_1, x_2 \mid (x_1,\dots,x_m) \in G\}\right)$ given by FLDM, if the decision-making variable $\left(x_3, x_4 \in G_2 = \{x_3, x_4 \mid (x_1,\dots,x_m) \in G\}\right)$ is the optimal solution of the SLDM, then $(x_1,\dots,x_m)$ is a feasible solution of (BLMOLSIQP) problem.

Definition 2. [13]

If $x_j^* \in R^n$, *(j=1,2,...,m)* is a feasible solution of the (BLMOLSIQP) problem (1) – (3); no other feasible solution $x_j \in G$ exists, such that $F_i(x_j^*) \leq F_i(x_j)$, *(i=1,2), (j=1,2,...,m)* so x_j^* is the optimal solution of the (BLMOLSIQP) problem.

3 α-Level Sets of Fuzzy Numbers or α-cut

α -cut or α -level set is one of the most important concepts to solve (BLMOLSIQP) problem with fuzzy numbers in the objective functions that convert fuzzy number form into equivalent deterministic form, but transformation process needs the following definitions to be known, for more details see [20]:

Definition 3.

Let R be the space of real numbers. A Fuzzy set $\widetilde{A}_i$ is a set of ordered pairs $\{(x, \mu_{\widetilde{A}_i}(x)) \mid x \in R\}$ where $\mu_{\widetilde{A}_i}(x)$:→ [0,1] is called membership function of fuzzy set.

Definition 4.

A convex fuzzy set, $\widetilde{A}_i$, is a fuzzy set in which: ∀ x, y ∈ R, ∀ λ ∈ [0, 1]

$$\mu_{\widetilde{A}_i}(\lambda x + (1 - \lambda) y) \geq \min [\mu_{\widetilde{A}_i}(x), \mu_{\widetilde{A}_i}(y)].$$

Definition 5.

A fuzzy number is a trapezoidal fuzzy number if the membership function of it is in the following form:

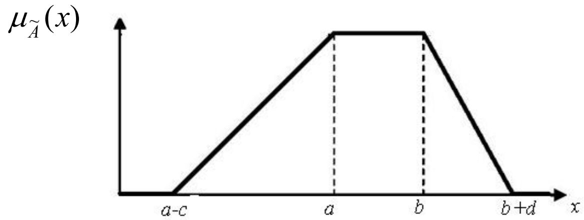


Fig. 1.1. Trapezoidal fuzzy number

Where *a* and *b* are respectively the lower and the upper bounds of the fuzzy number. We show any trapezoidal fuzzy number by $\tilde{A} = (a, b, c, d)$ where the support of $\tilde{A}$ is (*a-c, b+d*) and the model set of $\tilde{A}$ is [a, b].

Definition 6.

Trapezoidal fuzzy number (TFN) is a convex fuzzy set which is defined as $\tilde{A} = (x, \mu_{\tilde{A}_i}(x))$ where:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a-x}{a-c} & \text{if} \quad a - c \le x < a, \\ 1 & \text{if} \quad a \le x < b, \\ 1 - \dfrac{x-b}{b+d} & \text{if} \quad b < x \le b + d, \\ 0 & \text{otherwise} \ . \end{cases}$$

Definition 7.

The α –level set of a fuzzy set $\tilde{A}$ is a non-fuzzy set denoted by $(\tilde{A})_\alpha$ for which the degree of its membership functions exceeds or is equal to a real number

$\alpha \in [0, 1]$, i.e. $(\tilde{A})_\alpha = \{x \mid \mu_{\tilde{A}_i}(x) \ge \alpha\}$.

The α-level set of Ã is then; $(\tilde{A})_\alpha = \left[(\tilde{A})_\alpha^L, (\tilde{A})_\alpha^U \right]$ that is

$$(\tilde{A})_\alpha^L = (1-\alpha)a + \alpha\, b, \text{ and } (\tilde{A})_\alpha^U = (1-\alpha)d + \alpha c, \tag{4}$$

Where, $(\tilde{A})_\alpha^L$ and $(\tilde{A})_\alpha^U$ represent the lower and upper cuts respectively.

Since the (BLMOLSIQP) problem is maximization-type then replacing the fuzzy coefficient by their $(\tilde{A})_\alpha^U$ upper cuts, the problem (1) – (3), can be understood as the corresponding deterministic in the objective functions as follows:

[FLDM]

$$\underset{x_1, x_2}{Max} \quad F_1(x) = [\, f_{11}(x), \quad f_{12}(x), \ \ldots, f_{1n}(x) \,] \tag{5}$$

Where $x_3, \ldots, x_m$ solves

[SLDM]

$$\underset{x_3, x_4}{Max} \quad F_2(x) = [\, f_{21}(x), \quad f_{22}(x), \; \dots, \; f_{2n}(x) \,] \tag{6}$$

Where $x_5, \dots, x_m$ solves

Subject to
$x \in G.$

Such that $f_{ik} = A_{ik}\, x + \frac{1}{2} x^T L_{ik}\, x$ $(i = 1,2)$, $(k = 1,2,.., n)$.

Then, each level has its own optimal solution using Taylor's series, and decomposition algorithm together with weighted sum method.

4 Taylor Series Approach

It would be very complex to solve (BLMOLSIQP) problem using decomposition algorithm. So firstly, we transform the objective functions by using 1^{st} order Taylor series polynomial in the following form [19].

$$K_i(x) \cong F_i^{\wedge}(x) = F_i(x_j^*) + \sum_{j=1}^{m} (x_j - x_{ij}^*) \frac{\partial F_i(x_j^*)}{\partial x_j}, (j = 1,2,\dots,m), (i = 1,2) \tag{7}$$

Then we use the weighted sum method [11] to transform the objective functions in the upper level and lower level from multi-objective into single-objective, the weight of the first objective is greater than the weight of the second objective so the Bi-Level Large Scale Integer Linear Programming (BLLSILP) Problem can be written as:

[FLDM]

$$\underset{x_1, x_2}{Max}\, Q_1(x) = Max \sum_{j=1}^{m} c_{1j} x_j \tag{8}$$

Where $x_3, \dots, x_m$ solves

[SLDM]

$$\underset{x_3, x4}{Max}\, Q_2(x) = Max \sum_{j=1}^{m} c_{2j} x_j \tag{9}$$

Where $x_5, \dots, x_m$ solves

Subject to
$x \in G.$

5 Decomposition Algorithm for Bi Level Large Scale Integer Linear Programming Problem

To solve the (BLLSILP) problem by the decomposition algorithm [9], the FLDM gets the optimal solution using decomposition algorithm by breaking the large scale problem into n-sub problems that can be solved directly. Then, by inserting the FLDM decision variable to the SLDM to seek the optimal solution using the decomposition method.

5.1 The First-Level Decision-Maker (FLDM) problem

The FLDM problem of the (BLLSLIP) problem is as follows:

[FLDM]

$$\underset{x_1,x_2}{Max}\ Q_1(x) = Max \sum_{j=1}^{m} c_{1j} x_j \tag{10}$$

Subject to
$x \in G.$

The FLDM problem uses decomposition method [9] to solve the problem. If the FLDM doesn't have an integer optimal, then the FLDM will use Branch and Bound [10] to get an integer optimal solution.

5.2 The Second-Level Decision-Maker (SLDM) problem

Finally, according to the mechanism of the (BLLSLIP) problem, the First Level variables x_1^F, x_2^F should be passed to the Second-Level; so the second-level problem can be written as follows:

$$\underset{x_3,x_4}{Max}\ Q_2(x) = Max \sum_{j=1}^{m} c_{2j} x_j, \tag{11}$$

Subject to

$$(x_1^F, x_2^F, ..., x_m^F) \in G. \tag{12}$$

To obtain the optimal solution of the second level problem; the SLDM solves its master problem by the decomposition method [9] as the first level, If the SLDM doesn't have an integer optimal, then the FLDM will use Branch and Bound [10] to get an integer optimal solution.

Now the optimal solution ($x_1^F, x_2^F, x_3^S, x_4^S, x_5^S, ..., x_m^S$) of the SLDM is the optimal solution of the (BLLSLIP) problem.

6 Frank and Wolfe Algorithm [21]

This method deals with the following problem in which all constraints are linear:

$$Max\, Z = f(X), \tag{13}$$

Subject to
$AX \le b, X \ge 0.$

Let X^k be the feasible trial point at iteration $k.$ the objective function $f(X)$ can be expanded in the neighborhood of X^k using Taylor series. This gives

$$f(X) \cong f(X^k) + \partial f(X^k)(X - X^k) = (f(X^k) - \partial f(X^k)X^k) + \partial f(X^k)X. \tag{14}$$

The procedure calls for determining a feasible point $X = X^*$ such that $f(X)$ is maximized subject to the linear constraints of the problem. Because $f(X^k) - \partial f(X^k)X^k$ is a constant, the problem for determining X^* reduces for solving the linear program:

$$Max\ w_k(X) = \partial f(X^k)X, \tag{15}$$

Subject to
$AX \le b, X \ge 0.$

Given w_k is constructed from the gradient of $f(X)$ at X^k, an improved solution point can be secured if and only if $w_k(X^*) \succ w_k(X^k)$. From Taylor expansion, the condition does not achieve that $f(X^*) \succ f(X^k)$ unless X^* is in the neighborhood of X^k. However, given $w_k(X^*) \succ w_k(X^k)$, there must exist a point X^{k+1} on the line segment (X^k, X^*) such that $f(X^{k+1}) \succ f(X^k)$. The objective is to determine X^{k+1}. Define

$$X^{k+1} = (1-r)X^k + rX^* = X^k + r(X^* - X^k), 0 \prec r \le 1. \tag{16}$$

This means that X^{k+1} is a linear combination of X^k and X^*. Because X^k and X^* are two feasible points in a convex solution space, X^{k+1} is also feasible. The parameter r represents the step size.

The point X^{k+1} is determined such that $f(X)$ is maximized. Because X^{k+1} is a function of r only, X^{k+1} is determined by maximizing

$$h(r) = f(X^k + r(X^* - X^k)). \tag{17}$$

The procedure is repeated till it reaches the k-th iteration, $w_k(X^*) \le w_k(X^k)$ at this point, no further improvements are possible, and the process terminates with X^k as the best solution point.

7 An Algorithm for Solving (BLMOLSIQP) Problem with Fuzzy Numbers

A solution algorithm to solve (BLMOLSIQP) Problem, in which all decision parameters in the objective functions are symmetric trapezoidal fuzzy numbers, and have block angular structure of the constraints, is described in a series of steps as follows:

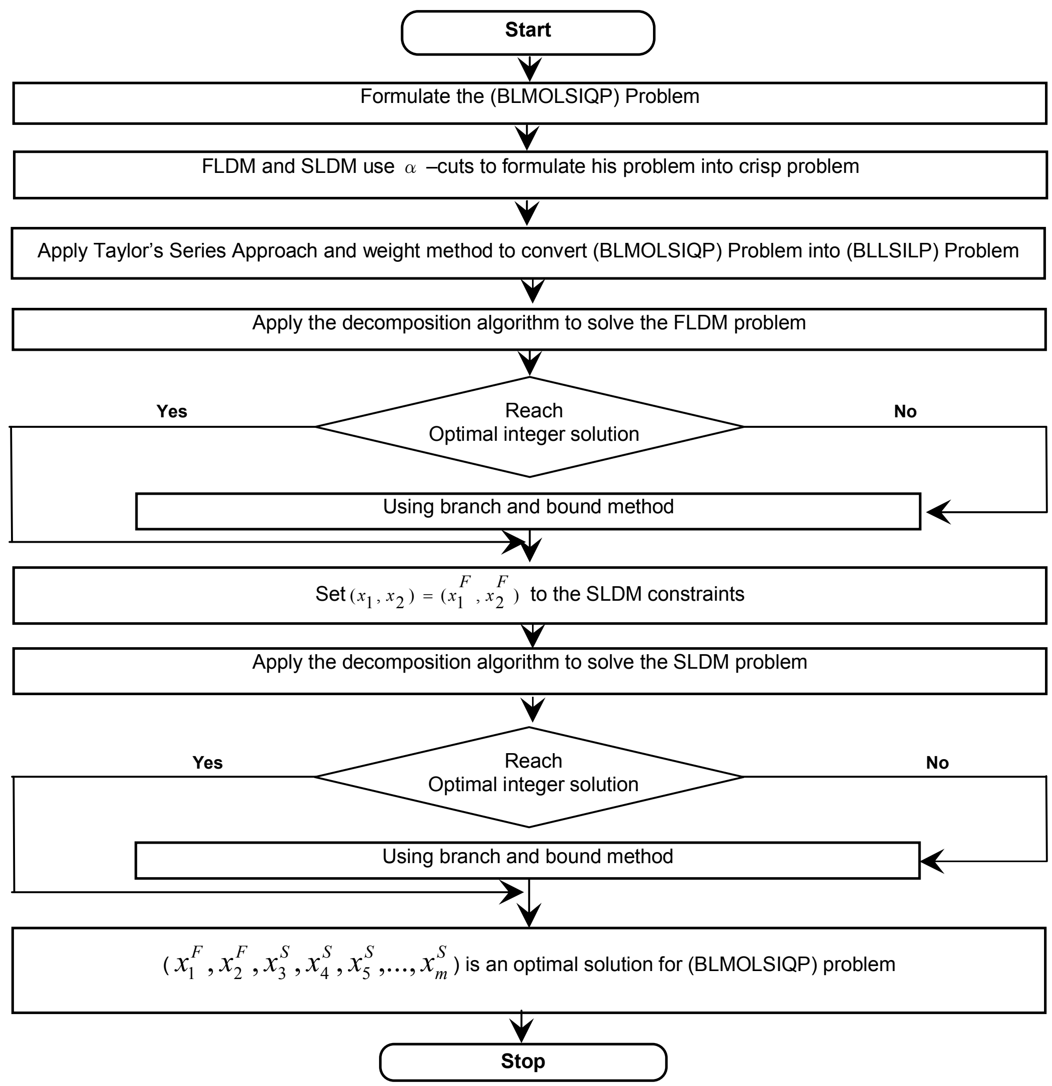
Step 1. Formulate the (BLMOLSIQP) Problem with fuzzy parameters in the objective functions.
Step 2. The FLDM and SLDM convert Problem (1) – (2) into Problem (5) – (6) by using α–cuts.
Step 3. Apply Taylor's series approach [19] to obtain polynomial objective function in Formula (7).
Step 4. Use weighted sum method [10] to obtain single objective and formulate the problem (8) – (9).
Step 5. Apply the decomposition algorithm [9] to solve the FLDM problem by breaking the large scale problems into n sub-problems that can be solved directly, then the optimal solution would be in (10).
Step 6. If the FLDM gets the optimal solution as an integer, go to Step 8.
Otherwise, go to Step 7.
Step 7. Using Branch and Bound method [11] to find integer optimal solution.

Step 8. Set $(x_1, x_2) = (x_1^F, x_2^F)$ to the SLDM constraints.
Step 9. Formulate the SLDM problem (11).
Step 10. Apply the decomposition algorithm [9] to solve the SLDM, then the optimal solution is obtained.
Step 11. If the SLDM`s optimal solution is an integer, go to Step 13.
Otherwise, go to Step 12.
Step 12. Use Branch and Bound method [10] to find integer optimal solution.
Step 13. $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^S, ..., x_m^S)$ is an optimal solution for (BLMOLSIQP) problem.
Step 14. Stop.

Remark 1. For (BLMOLSIQP) problem, the Lingo package is suggested as a basic solution tool.

8 A Flowchart for Solving (BLMOLSIQP) Problem

A flowchart to explain the suggested algorithm is described as follows:



9 Numerical Example

To demonstrate the solution for (BLMOLSIQP) problem with fuzzy numbers, let us consider the following problem:

[FLDM]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}\begin{Bmatrix} ((2,3,1,1)x_1^2 + 2x_2 + (1,3,1,1)x_2^2 + x_3 + 6x_5^2 + (7,9,3,3)x_6), \\ ((3,6,2,2)x_1 + (2,4,1,1)x_1^2 + (4,6,3,3)x_2 + 8x_2^2 + 2x_3 + (1,3,1,1)x_4^2 + x_5 + 2x_6^2), \\ ((x_1 + (5,7,2,2)x_1^2 + (8,10,3,3)x_2 + (1,2,1,1)x_3^2 + x_4 + (2,4,1,1)x_5^2) \end{Bmatrix}$$

Where x_3, x_4, x_5, x_6 solves

[SLDM]

$$\underset{x_3,x_4}{Max}F_2(x) = \underset{x_3,x_4}{Max}\begin{Bmatrix} (x_1 + (1,2,1,1)x_2^2 + 5x_3^2 + (3,5,2,2)x_4 + (4,6,3,3)x_4^2 + 3x_6 + x_6^2), \\ (2x_1 + (1,3,1,1)x_1^2 + 3x_2 + 4x_3 + (7,8,4,4)x_3^2 + (3,5,2,2)x_4 + (4,6,3,3)x_4^2 + 6x_5^2 + x_6), \\ (x_1^2 + x_2 + 5x_2^2 + (10,12,8,8)x_3 + 6x_4 + (6,8,4,4)x_4^2 + x_5 + (1,3,1,1)x_6) \end{Bmatrix}$$

Where x_5, x_6 solves

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 90,$

$2x_1 - x_2 \le 10,$

$x_1 + 3x_2 \le 35,$

$-x_3 + x_4 \le 50,$

$2x_5 + x_6 \le 40,$

$x_5 + 2x_6 \le 60,$

$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$ and integer.

Firstly, Applying α –Cuts [20] to transform the fuzzy number form in to equivalent crisp form Let α = 0.5to compute $(\tilde{A})_{0.5}^{U}$ so, the problem reduces to

[FLDM]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}\begin{Bmatrix} (3.5x_1^2 + 2x_2 + 3.5x_2^2 + x_3 + 6x_5^2 + 10.5x_6), \\ (7x_1 + 4.5x_1^2 + 7.5x_2 + 8x_2^2 + 2x_3 + 3.5x_4^2 + x_5 + 2x_6^2), \\ (x_1 + 8x_1^2 + 11.5x_2 + 2.5x_3^2 + x_4 + 4.5x_5^2) \end{Bmatrix}$$

Where x_3, x_4, x_5, x_6 solves

[SLDM]

$$\underset{x_3,x_4}{Max}\ F_2(x) = \underset{x_3,x_4}{Max}\begin{Bmatrix} (x_1 + 2.5x_2^2 + 5x_3^2 + 6x_4 + 7.5x_4^2 + 3x_6 + x_6^2), \\ (2x_1 + 3.5x_1^2 + 3x_2 + 4x_3 + 10x_3^2 + 6x_4 + 7.5x_4^2 + 6x_5^2 + x_6), \\ (x_1^2 + x_2 + 5x_2^2 + 16x_3 + 6x_4 + 10x_4^2 + x_5 + 3.5x_6) \end{Bmatrix}$$

Where x_5, x_6 solves

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 90,$

$2x_1 - x_2 \leq 10,$

$x_1 + 3x_2 \leq 35,$

$- x_3 + x_4 \leq 50,$

$2x_5 + x_6 \leq 40,$

$x_5 + 2x_6 \leq 60,$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.$ and integer.

Secondly, applying the first order Taylor series [19] to convert the quadratic objectives functions to linear objectives functions and use the weighting method [10] to convert multi objectives to single objective so the (BLLSLIP) problem is written as follows:

[FLDM]

$$\underset{x_1, x_2}{Max}\ F_1(x) = \underset{x_1, x_2}{Max} \begin{Bmatrix} (7x_1 + 9x_2 + x_3 + 12x_5 + 10.5x_6 - 13), \\ (16x_1 + 23.5x_2 + 2x_3 + 7x_4 + x_5 + 4x_6 - 18), \\ (17x_1 + 11.5x_2 + 5x_3 + x_4 + 9x_5 - 15) \end{Bmatrix}$$

Where x_3, x_4, x_5, x_6 solves

[SLDM]

$$\underset{x_3, x_4}{Max}\ F_2(x) = \underset{x_3, x_4}{Max} \begin{Bmatrix} (x_1 + 5x_2 + 10x_3 + 21x_4 + 5x_6 - 16), \\ (9x_1 + 3x_2 + 24x_3 + 21x_4 + 12x_5 + x_6 - 27), \\ (2x_1 + 11x_2 + 16x_3 + 26x_4 + x_5 + 3.5x_6 - 10) \end{Bmatrix}$$

Where x_5, x_6 solves

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 90,$

$2x_1 - x_2 \leq 10,$

$x_1 + 3x_2 \leq 35,$

$- x_3 + x_4 \leq 50,$

$2x_5 + x_6 \leq 40,$

$x_5 + 2x_6 \leq 60,$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.$ and integer.

The objective functions of the FLDM are transformed by Weight Method [10] to a single objective as follows:

$$\underset{x_1, x_2}{Max}\ F_1(x) = \underset{x_1, x_2}{Max}(13.33x_1 + 14.66x_2 + 2.66x_3 + 2.66x_4 + 7.33x_5 + 4.83x_6 - 15.33)$$

Subject to

$x \in G.$

Where w_1=w_2=w_3=0.333333; w_1+w_2+w_3 = 1.

After four iterations the first level decision maker optimal solution is obtained:

$$(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F) = (9.2857, 8.5714, 52.1428, 0, 20, 0).$$

So, $F_1 = 519.8528$

The FLDM uses Branch and Bound to get integer optimal solution then we get

$$(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F) = (8, 9, 2, 51, 20, 0).$$

So, $F_1 = 511.2822$

Then take the first level decision maker solution and set $(x_1^F, x_2^F) = (8,9)$ to the second level constraint.

$$\underset{x_3,x_4}{Max} F_2(x) = \underset{x_3,x_4}{Max}(3.99x_1 + 6.33x_2 + 16.66x_3 + 22.66x_4 + 4.33x_5 + 3.166x_6 - 17.66)$$

Subject to
$x \in G.$

The SLDM will repeat the same steps as the first level decision maker until the second level decision maker obtains the optimal solution:

$$(x_3^s, x_4^s, x_5^s, x_6^s) = (11.5, 61.5, 0, 0).$$

So, $F_2 = 1656.998$

The SLDM uses Branch and Bound to get integer optimal solution, then we get

$$(x_1^F, x_2^F, x_3^s, x_4^s, x_5^s, x_6^s) = (8, 9, 12, 61, 0, 0).$$

So, $F^*_1 = 511.2822$, $F^*_2 = 1653.998$

Also, we can apply Frank and Wolfe algorithm [21] to convert the quadratic objectives functions to linear objectives functions and use the weighting method [10] to convert multi objectives to single objective so the solution of the FLDM using Weighting Method, Frank and Wolfe algorithm, and Decomposition Algorithm=5294.453 .

$$(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F) = (9.285, 8.571, 1.071, 51.071, 20, 0).$$

The FLDM uses Branch and Bound to get integer optimal solution then we get

$$(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F) = (8, 8, 2, 52, 20, 0).$$

So, $F_1 = 5247.475$

Solution of the SLDM using weighted sum method, Frank and Wolfe algorithm, Decomposition Algorithm, and FLDM decision variables $(x_1^F, x_2^F) = (8,8)$ = 33476.65

$$(x_1^F, x_2^F, x_3^S, x_4^S, x_5^S, x_6^S) = (8,8,12,62,0,0).$$

So, $F^*_1 = 5247.475$, $F^*_2 = 33476.65$

Table 1 compares the results of applying Frank algorithm and Taylor series for solving the (BLMOLSIQP) problem with fuzzy numbers in the objective functions.

Table 1. Comparison between results of applying Frank Algorithm and Taylor Series

Level	The results using Taylor	The results using Frank and Wolfe
FLDM	511.2822	5247.475
SLDM	1653.998	33476.65

The Taylor algorithm produces approximated, inaccurate, but fast solutions. These solutions can be used in fields such as agricultural decisions.

The Frank and Wolfe algorithm introduces accurate but slow solutions. These solutions can serve in fields such as medical and financial decisions.

Finally, in comparing between the result found in O. Emam et al. [22] and the proposed algorithm, the result shows that the proposed algorithm better than the result found in O. Emam et al. [22]. The table below introduces the following:

Table 2. Comparison between the result found in Emam et al. (2017) [22] and the proposed algorithm

Level	The result found in O. Emam et al. (2017)[22]	The proposed algorithm
FLDM	671.748	511.2822
SLDM	2132.10	1653.998

10 Conclusion and Future Points

This paper suggested an algorithm to solve the **(**BLMOLSIQP**)** problem with fuzzy parameters in the objective functions at every level to be maximized. The suggested algorithm has used α-Level sets of fuzzy numbers, weighted sum method, then all decision makers attempt to optimize their problems separately as a large scale quadratic programming using Dantzig and Wolfe decomposition method and Taylor's series together with constraint method. Then, compared the proposed algorithm to Frank and Wolfe algorithm to demonstrate its effectiveness. Finally, a numerical example was given to clarify the main results developed in this paper.

However, there are many other aspects which should by explored and studied in the area of fuzzy large scale optimization such as:

1- Multi-level Multi-objective large scale integer quadratic programming problem with fuzzy parameters in the constraints.
2- Bi-level Multi-objective large scale integer quadratic programming problem with fuzzy parameters in the objective functions and constraints.

3- Multi-level Multi-objective large scale integer quadratic programming problem with fuzzy parameters in the objective functions and constraints.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Abou-El-Enien THM. On the solution of a special type of large scale integer linear vector optimization problems with uncertain data through TOPSIS approach. International Journal of Contemporary Mathematical Sciences. 2011;6:657–669.

[2] Barkha S, Rajendra D. Optimum solution of fuzzy linear programming problem for trapezoidal number. VSRD-TNTJ. 2012;3(7):268-276.

[3] Mahdavi N, Nasseri SH. Duality in fuzzy number linear programming by use of a certain linear ranking function. Applied Mathematics and Computation. 2006;180:206-216.

[4] Nasseri SH, Adril E, Yazdani A, Zaefarian R. Simplex method for solving linear programming problems with fuzzy numbers. World Academy of Science , Engineering and Technology. 2005;10: 285-288.

[5] Youness EA, Emam OE, Hafez MS. Simplex method for solving Bi-level linear fractiopnal integer programming problems with fuzzy numbers. International Journal of Mathematical Sciences and Engineering Applications. 2013;3:351-363.

[6] Youness EA, Emam OE, Hafez MS. Bi-level multi-objective fractional integer programming. Applied Mathematics & Information. 2014;6:2857-2863.

[7] Sultan TI, Emam OE, Abohany AA. A decomposition algorithm for solving a three–levellarge scale linear programming problem. Applied Mathematics and Information Science. 2014;5:2217-2223.

[8] Abo-Sinna MA, Abou-El-Enien THM. An interactive algorithm for large scale multiple objective programming problems with fuzzy parameters through topsis approach. Yugoslav Journal of Operations Research. 2011;21:253-273.

[9] Dantzig G, Wolfe P. The decomposition algorithm for linear programming. Econometric. 1961;9(4): 767–778.

[10] All-integer programming algorithm, in: Muth, J.F. and Thompson, G.L.(Eds.), Industrial Scheduling, Prentice-Hall, Englewood Cliffs, N.J, Gomory, R.E.; 1958.

[11] Taha HT. Operation research-an introduction. 6th Edition, Mac Milan Publishing Co, New York; 1997.

[12] Emam OE. A fuzzy approach for bi-level integer nonlinear programming problem. Applied Mathematics and Computation. 2006;172:62–71.

[13] Emam OE. Interactive approach to bi-level integer multi-objective fractional programming problem. Applied Mathematics and Computation. 2013;233:17-24.

[14] Baky IA. Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach. Applied Mathematical Modeling. 2010;34:2377-2387.

[15] Abo-Sinna MA, Abou-El-Enien THM. An interactive algorithm for large scale multiple objective programming problems with fuzzy parameters through TOPSIS approach. Applied Mathematics and Computation. 2006;177:515-527.

[16] Osman MS, Saad OM, Hasan AG. Solving special class of large scale fuzzy multi objective integer linear programming problems. Fuzzy Sets and Systems. 1999;107:289-297.

[17] Emam OE, Fathy E, Abohany AA. An interactive model for fully rough three level large scale integer linear programming problem. International Journal of Computer Applications. 2016;155(11):1-12.

[18] Emam OE, Fathy E, Helmy MA. Fully fuzzy multi-level linear programming problem. International Journal of Computer Applications. 2016;155(7):18-26.

[19] Saad OM, Mohamed TR, Alshafae MK, Abdellah EF. Taylor series approach for solving chance-constrain multiobjective integer linear fractional programming problem. International Journal of Mathematical Archive. 2012;3:18 – 23.

[20] Sakawa M. Fuzzy sets and interactive multi-objectiveoptimization. Plenum Press, New York, NY, USA; 1993.

[21] Frank, Wolfe P. An algorithm for quadratic programming. Naval Research Logistics Quarterly. 1956; 3:95–110.

[22] Emam OE, Fathy E, Abdullah AA. On fuzzy Bi-level multi-objective large scale integer quadratic programming problem. International Journal of Computer Applications. 2017;159(2):28-33.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/24259