Journal of Advances in Mathematics and Computer Science

27(2): 1-14, 2018; Article no.JAMCS.39203

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Multidisemigraph Poset

Prabhakar R. Hampiholi[1], Meenal M. Kaliwal[2*], Shailaja S. Shirkol[3] and Jotiba P. Kitturkar[4]

[1]Department of Mathematics, Gogte Institute of Technology, Belagavi, Karnataka, India.
[2]Department of Mathematics, KLS Vishwanathrao Deshpande Rural Institute of Technology, Haliyal, Karnataka, India.
[3]Department of Mathematics, SDM College of Engineering and Technology, Dharwad, Karnataka, India.
[4]Department of Mathematics, Maratha Mandal Engineering College, Belagavi, Karnataka, India.

Authors' contributions

This work was carried out in collaboration between all authors. Authors PRH and MMK wrote the first draft of the manuscript. Authors SSS and JPK suggested the hierarchy of the paper and managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2018/39203
Editor(s):
(1) Dariusz Jacek Jakóbczak, Assistant Professor, Chair of Computer Science and Management in this Department, Technical University of Koszalin, Poland.
Reviewers:
(1) Pasupuleti Venkata Siva Kumar, VNR Vignana Jyothi Institute of Engineering and Technology, India.
(2) Sheetal Naresh Ghorpade, RMD Sinhgad School of Engineering, India.
Complete Peer review History: http://www.sciencedomain.org/review-history/24279

Original Research Article

Received: 3rd January 2018
Accepted: 17th April 2018
Published: 23rd April 2018

Abstract

A semigraph is a generalization of a graph and is introduced by E. Sampathkumar. In this paper, the concept of multidisemigraph, multidisemigraph poset (MDSPOSET) and various relations for a multidisemigraphs are defined with respect to the middle vertices and the end vertices. Also, the $n-$ dimensional co-ordinate system is represented by using the concept of MDSPOSET.

Keywords: Disemigraph; multidisemigraph; MDSPOSET.

1 Introduction

Semigraph [1] is considered as a natural generalization of a graph as it resembles a graph when drawn in a plane. Currently, extensive research is being carried out in various concepts

**Corresponding author: E-mail: mmkmeenal@gmail.com;*

of semigraphs such as domination [2,3], matrix theory, enumeration of semigraphs [4,5], disemigraph poset [6].

Analogous to digraphs [7,8] the concept of Disemigraph was first introduced in [1], in which several variants like an arc, subarc, partial arc, subdisemgraph, spanning subdisemigraph, walk, path, trail, circuits, cycles, connected disemigraphs, Eulerian disemigraphs are defined and characterized. E. Sampathkumar and L. Pushpalatha have defined the matrix of a semigraph [9] and also the characterization of a matrix of a disemigraph is stated (unpublished work). Recently, another parameter a po-disemigraph of disemigraph is introduced [10].

In this paper, multidisemigraph poset (MDSPOSET) is defined and represented by a relation matrix. In [1] only symmetric relation for a disemigraph is defined, whereas this paper defines various types of relations, considering the adjacent properties of middle vertices, end vertices and $(m,e)-$ vertices.

2 Basic Definitions

Definition 2.1: [1] A digraph D consists of a finite set V of points and a set of collection of ordered pairs of distinct points u and v. Such a pair $\{u,v\}$ is called an arc which is directed from u to v. The arc is incident with u and v.

Example 2.1: Consider the digraph D with the vertex set $V(D)=\{v_1,v_2,v_3,v_4,v_5\}$ and arc set $E(D)=\{(v_1),(v_1,v_2),(v_2,v_1),(v_1,v_3),(v_3,v_4),(v_2,v_4),(v_5,v_4),(v_5,v_2)\}$ as shown in Fig. 2.1. Note that the vertex v_1 contains a loop.

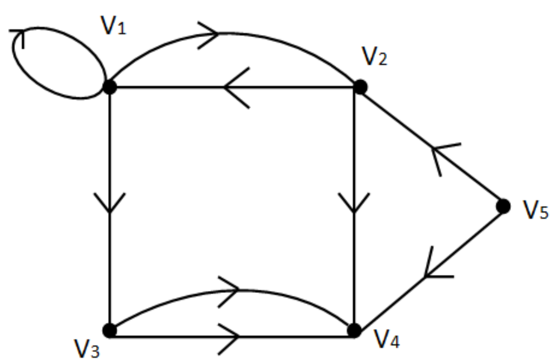

Fig. 2.1. Digraph D

Definition 2.2: [11] For a given non-empty set L, a binary relation on L, is said to be a partial order if,

(i) $x \le x$
(ii) $x \le y$ and $y \le x \to x = y$ (antisymmetry)
(iii) $x \le y$ and $y \le z$ implies $x \le z$ (transitivity) for every $x, y, z \in L$.

A set L with a partial order relation $\le$ is called a partially ordered set or a poset and is denoted by $(L,\le)$.

Definition 2.3: [11] Let $(L,\le)$ be a poset and $A \subseteq L$ then,

(i) An element $x \in L$ is a least upper bound or a supremum of A if x is an upper bound of A and $x \le y$, where y is an upper bound of A.

(ii) An element $x \in L$ is a greatest lower bound or an infimum of A, if x is a lower bound of A and $x \geq y$, where y is any lower bound of A.

The above definition of the supremum and infimum is illustrated by the following example.

Example 2.2: Let $A = \{2,3,6,12,24,36\}$ and the relation '$\leq$' be such that $x \leq y$ if x divides y. Clearly, $(A,\leq)$ is a poset. The Hasse diagram of $(A,\leq)$ is shown in Fig. 2.2. The arrow marks will not be used in the figure with the convention that xRy is considered in the upward direction. (For example, $2R6$ but 6 is not related to 2).

For the subset $B = \{2,3,6\}$ the upper bounds are $6,12, 24,36$ and B has no lower bound. Also, $sup\,B = 6$, while there is no greatest lower bound of B or $\inf B$. Also, for the subset $C = \{6,12\}$, the least upper bound of C = sup C = 12 and greatest lower bound of C = inf C = 6.

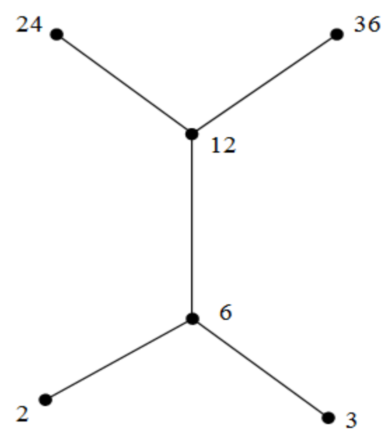

Fig. 2.2. Hasse diagram

Definition 2.4: [11] A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice.

Definition 2.5: [3] A semigraph $G = (V, X)$ is an ordered pair, where V is a non-empty set of elements called vertices of G, and X is a set of $n-$ tuples, called edges of G, of distinct vertices for $n \geq 2$, satisfying the following conditions:

SG1: Any two edges have atmost one vertex in common.

SG2: Two edges $(u_1, u_2, u_3, ..., u_n)$ and $(v_1, v_2, v_3, ..., v_m)$ are considered to be equal if and only if, (i) $m = n$ and (ii) either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Thus, the edge $E = (u_1, u_2, u_3, ..., u_n)$ is same as the edge $(u_n, u_{n-1}, ..., u_1)$. The vertices u_1 and u_n are called the end vertices of E, while $u_2, u_3, ..., u_{n-1}$ are called the middle vertices of E. Also, a vertex which is a middle vertex for one edge and the end vertex of another edge is said to be a middle-cum-end vertex or a (m,e) vertex.

Example 2.3: Consider a semigraph $G=(V,X)$ with vertex set $V=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9\}$ and the edge set $X=\{(v\ ,v_2,v_3),(v_3,v_4,v_5,v_6),(v_6,v_7),(v_1,v_8,v_7),(v_1,v_6),(v_7,v_4)\}$ respectively. The vertices v_2,v_5 and v_8 are the middle vertices and the vertex v_4 is a (m,e) vertex. The vertex v_9 is an isolated vertex.

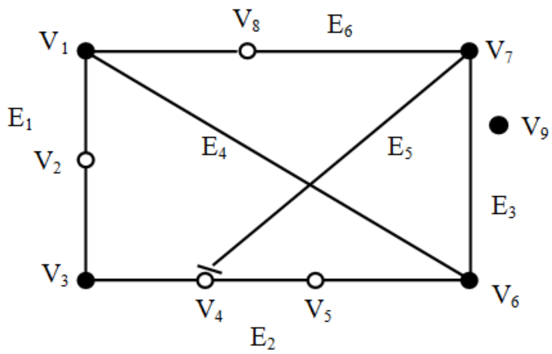

Fig. 2.3. Semigraph

Definition 2.6: [3] A disemigraph or a directed semigraph $\vec{D}=(V,X)$, is a finite set of objects called vertices together with a (possibly empty) set of ordered $n-$ tuples of distinct vertices of $\vec{D}$ for various $n\geq 2$, called directed edges or arcs, satisfying the condition:

For any two distinct vertices u and v in a disemigraph $\vec{D}$ there is at most one arc containing u and v such that u is adjacent to v and at most one arc containing u and v such that v is adjacent to u.

In other words a directed semigraph or a disemigraph $\vec{D}$ is a semigraph whose edges are directed or oriented. These directed edges are called arcs of $\vec{D}$ and are denoted by $\vec{e}$.

Definition 2.6 of a disemigraph is illustrated by the following example.

Example 2.4: Let us consider a disemigraph $\vec{D}=(V,X)$, whose vertex set $V=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$ and the arcs of $\vec{D}=(V,X)$ are $\vec{e_1}=(v_1,v_2,v_3,v_4,v_5)$, $\vec{e_2}=(v_7,v_6,v_2)$, $\vec{e_3}=(v_8,v_3)$ and $\vec{e_4}=(v_7,v_8,v_5)$. Note that in a semigraph, the edge (v_1,v_2,v_3,v_4,v_5) is same as the edge (v_5,v_4,v_3,v_2,v_1). But, in a disemigraph, if (v_1,v_2,v_3,v_4,v_5) is an arc then, (v_5,v_4,v_3,v_2,v_1) need not be an arc. Further, if in case, such arcs exist, they will be treated as different arcs.

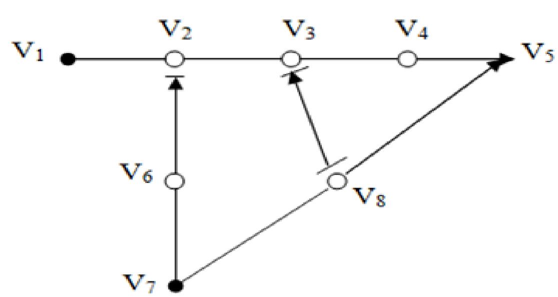

Fig. 2.4. Disemigraph $\vec{D}$

3 Results and Discussion

In this section, the concept of multidisemigraph is introduced and few terminology of relations related to multidisemigraph are defined and investigated.

Definition 3.1: A multidisemigraph is a disemigraph which contains parallel edges and self loops.

Therefore, any two edges of the multidisemigraph can have more than two vertices in common.

Example 3.1: Let $\overrightarrow{D}=(V,X)$ be a multidisemigraph poset with vertex set $V=\{v_1,v_2,v_3,v_4,v_5\}$ and edge set $X=\{(v_1),(v_1,v_2,v_3),(v_3,v_2,v_1),(v_2,v_4),(v_4,v_5,v_3),(v_5,v_2)\}$. Here, the edge $\{v_1,v_2,v_3\}$ contains parallel edges $\{v_1v_2,v_2v_1,v_2v_3,v_3v_2\}$. Also the vertex v_1 contains a loop.

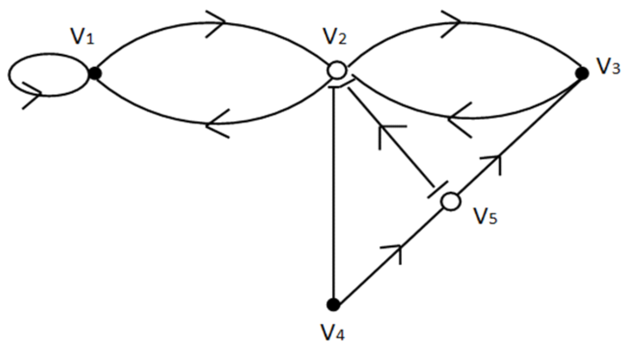

Fig. 3.1. Multidisemigraph

Definition 3.2: Let $\overrightarrow{D}=(V,X)$ be a multidisemigraph. Let $V=\{v_1,v_2,...,v_n\}$ and $X=\{E_1,E_2,...,E_k\}$ be the vertex set and the edge set of $\overrightarrow{D}$ respectively. A binary relation $\Re$ is said to exist, if $v_i\Re v_j$. The relation $\Re$ is either an adjacency relation or consecutive adjacency relation.

The relation $\Re$ of a disemigraph is categorized into three types.

Definition 3.3: A binary relation on a multidisemigraph $\overrightarrow{D}=(V,X)$ is said to be an $em-$relation if for every pair of vertices (v_i,v_j), there is a directed edge from v_i to v_j, where v_i is a terminal vertex and v_j is a middle vertex. The $em-$relation is denoted by $\Re_{em}$.

Example 3.2: In Fig. 3.2, we observe that $v_1\Re_{em}v_3$ and $v_1\Re_{em}v_4$.

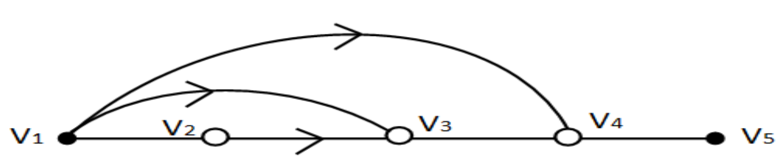

Fig. 3.2. $\Re_{em}$ relation

Definition 3.4: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be a $me-$relation relation if for every pair of vertices (v_i, v_j), there is a directed edge from v_i to v_j, where v_i is a middle vertex and v_j is the terminal vertex. The $me-$relation is denoted by $\Re_{me}$.

Example 3.3: In Fig. 3.3, we observe that $v_2\Re_{me}v_1$, $v_3\Re_{me}v_1$, $v_4\Re_{me}v_1$, and $v_3\Re_{me}v_5$.

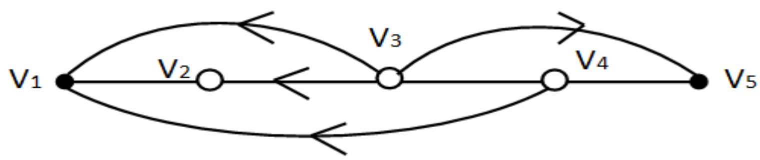

Fig. 3.3. $\Re_{me}$ **relation**

Definition 3.5: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be an $ee-$relation if for every pair of terminal vertices v_i and v_j,

(i) There is a directed edge from v_i to v_j or
(ii) There is a directed edge from v_j to v_i or
(iii) v_i or v_j may contain loops.

The $me-$relation is denoted by $\Re_{ee}$.

Example 3.4: In Fig. 3.4, we observe that $v_1\Re_{ee}v_1$, $v_1\Re_{ee}v_5$, $v_5\Re_{ee}v_1$ and $v_5\Re_{ee}v_5$.

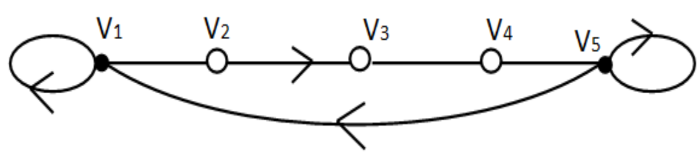

Fig. 3.4. $\Re_{ee}$ **relation**

Definition 3.6: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be a $mm-$relation if for every pair of middle vertices v_i and v_j,

(i) There is a directed edge from v_i to v_j or
(ii) There is a directed edge from v_j to v_i or
(iii) v_i or v_j may contain loops.

The $mm-$relation is denoted by $\Re_{mm}$.

Example 3.5: In Fig. 3.5, we observe that $v_2\Re_{mm}v_2$, $v_2\Re_{mm}v_3$, $v_2\Re_{mm}v_4$, $v_3\Re_{mm}v_4$ and $v_4\Re_{mm}v_4$.

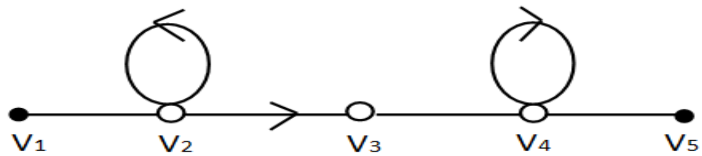

Fig. 3.5. $\Re_{mm}$ relation

Definition 3.7: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be an $ee-$reflexive if for every end vertex $v_i \in V_e$, where V_e is the set of end vertices, there exists a partial arc or subarc from v_i to itself such that $v_i\Re_{ee}v_i$. The $ee-$ reflexive is denoted by $\Re_{ee}$.

Definition 3.8: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be a $mm-$reflexive if for every middle vertex $v_i \in V_m$, where V_m is the set of middle vertices, there exists a partial arc or subarc from v_i to itself such that $v_i\Re_{mm}v_i$. The $mm-$reflexive is denoted by $\Re_{mm}$.

Here, we note that the $e-$reflexive and the $m-$reflexive relations generate loops for the end vertices and the middle vertices respectively.

Remark 3.9: $\Re_{em}$ and $\Re_{me}$ relations does not hold good for the reflexive property for multidisemigraphs.

Definition 3.10: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $ee-$irreflexive if for every end vertex $v_i \in V_e$, where V_e is the set of end vertices, there does not exist a partial arc or subarc from v_i to itself such that $v_i \notin \Re_{ee}$. The $ee-$reflexive is denoted by $\Re_{ee}$.

Definition 3.11: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $mm-$irreflexive if for every middle vertex $v_i \in V_m$ where V_m, is the set of middle vertices, there does not exist a partial arc or subarc from v_i to itself such that $v_i \notin \Re_{mm}$. The $mm-$reflexive is denoted by $\Re_{mm}$.

Definition 3.12: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $ee-$transitive if (v_i,v_j) and (v_j,v_k) are arcs in $\vec{D}$ then, (v_i,v_k) is also an arc $\vec{D}$, where v_i,v_j and v_k are the end vertices. The $ee-$reflexive is denoted by $\Re_{ee}$.

Example 3.6: In Fig. 3.6, the arcs $(v_1,v_3)\in\vec{D}$ and $(v_3,v_5)\in\vec{D}$, therefore the arc $(v_1,v_5)\in\vec{D}$. Thus $v_1\Re_{ee}v_5$.

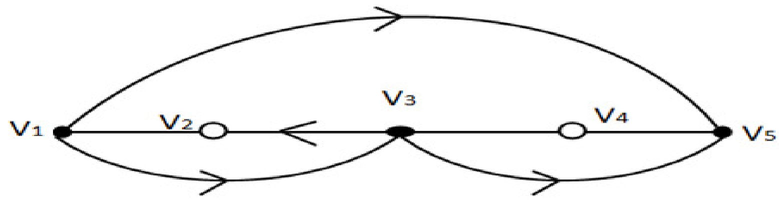

Fig. 3.6. $\Re_{ee}$ **transitivity**

Definition 3.13: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $mm-$ transitive if (v_i,v_j) and (v_j,v_k) are partial arcs or subarcs in $\vec{D}$ then, (v_i,v_k) is also an arc $\vec{D}$, where v_i, v_j and v_k are the middle vertices. The $mm-$ reflexive is denoted by $\Re_{mm}$.

Example 3.7: In Fig. 3.7, the arc $(v_2,v_3)\in\vec{D}$ and $(v_3,v_4)\in\vec{D}$, therefore the arc $(v_2,v_4)\in\vec{D}$. Thus, $v_2\Re_{mm}v_4$.

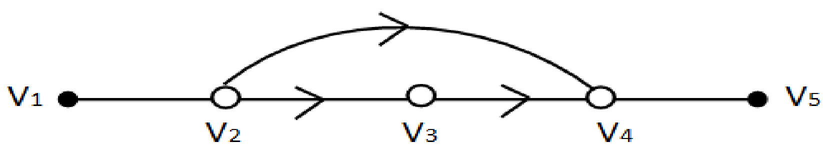

Fig. 3.7. $\Re_{mm}$ **transitivity**

Here, we note that $\Re_{ee}$ transitive and $\Re_{mm}$ transitive relations generate parallel edges for the pair of end vertices and middle vertices respectively.

Definition 3.14: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $ee-$ symmetric if $(v_i,v_j)\in\vec{D}$ then, $(v_j,v_i)\in\vec{D}$, where both $v_i,v_j\in V_e$. The $ee-$ symmetric is denoted by $\Re_{ee}$.

Definition 3.15: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $mm-$ symmetric if $(v_i,v_j)\in\vec{D}$ then $(v_j,v_i)\in\vec{D}$, where both $v_i,v_j\in V_m$. The $mm-$ symmetric is denoted by $\Re_{mm}$.

Definition 3.16 A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $ee-$ antisymmetric if $(v_i,v_j)\in\vec{D}$ and $(v_j,v_i)\in\vec{D}$ then $v_i=v_j$, where $v_i,v_j\in V_e$. The $ee-$ antisymmetric is denoted by $\Re_{ee}$.

Definition 3.17: A binary relation on a multidisemigraph $\vec{D}=(V,X)$ is said to be $mm-$ antisymmetric if $(v_i,v_j)\in\vec{D}$ and $(v_j,v_i)\in\vec{D}$ then $v_i=v_j$, where both $v_i,v_j\in V_m$. The $mm-$ antisymmetric is denoted by $\Re_{mm}$.

Remark 3.18: $\Re_{em}$ and $\Re_{me}$ relations does not hold good for transitive, symmetric and antisymmetric property for multidisemigraphs.

We prove the following theorem on the transitive relation of the multidisemigraph.

Theorem 3.19: A relation $\Re_{ee}$ (or $\Re_{mm}$) is $\Re_{ee}$ transitive (or $\Re_{mm}$ transitive) if and only if there is a weak path ($w-$ path) of length greater than one, from vertex v_i to vertex v_k , $\forall(v_i,v_j),(v_j,v_k)$ paths.

Proof: Suppose $\Re_{ee}$ (or $\Re_{mm}$) exists for a multidisemigraph $\vec{D}$ and let there exists a $w-$path of length greater than one from vertex v_i to vertex v_k then, this implies that there exist two consecutive maximal subarcs containing vertex v_j such that $(v_i,v_j)\in\vec{D}$ and (v_j,v_k) forms the partial subarcs. Since the relation $\Re_{ee}$ (or $\Re_{mm}$) is transitive there exists a maximal subarc from v_i to v_k . Hence there is a $w-$ path of length greater than one from vertex v_i to vertex v_k .

Conversely, suppose there exist $w-$ paths of length at least one from v_i to v_j and from v_j to v_k . This implies that, there exists at least one vertex v_j such that $(v_i,v_j)\in\vec{D}$ and (v_j,v_k) forms the partial subarcs. Hence there exists a partial subarc from v_i to v_k . This proves that $\Re_{ee}$ (or $\Re_{mm}$) relation exists for multidisemigraph $\vec{D}$.

Definition 3.20: Let $MD=(V,X)$ be a multidisemigraph. Then the relation $\Re$ on the vertices of $MD=(V,X)$ is a partially ordered set (POSET) if the following conditions hold:

(i) $(a,a)\in\Re$, $\forall a\in V$ (reflexive),
(ii) $(a,b)\in\Re$ and $(b,a)\in\Re$ if and only if $a=b$ (antisymmetric) and
(iii) $(a,b),(b,c)\in\Re\Rightarrow(a,c)\in\Re$ (transitive).

Example 3.4: Let us consider a poset, $A=\{1,2,3,4,6,9,12,18,36\}$ on the set of all positive divisors of 36, by the relation $\Re$ of divisibility such that $a\Re b$ if and only if a divides of b .

$$\Re=\left\{\begin{matrix}(1,1),(2,2),(3,3),(4,4),(6,6),(9,9),(12,12),(18,18),(36,36),(1,2),(1,3),\\(1,4),(1,6),(1,9),(1,12),(1,18),(1,36),(2,4),(2,6),(2,12),(2,18),(2,36),\\(3,6),(3,9),(3,12),(3,18),(3,36),(4,12),(4,36),(6,12),(6,18),(6,36),\end{matrix}\right\}$$

By using the structure of a semigraph as shown in Fig. 3.8, clearly we can identify that the element 1 is related to all the elements of the set $A=\{1,2,3,4,6,9,12,18,36\}$.

Remark 3.21: A relation $\Re$, in terms of multidisemigraph poset or MDSPOSET,

i) The Infimum and Supremum are both represented by the end vertices.
ii) The remaining elements of the set A are denoted by middle vertices.

Theorem 3.22: A multidisemigraph is a poset (MDSPOSET) with respect to a relation $\Re$, if and only if the row and column entries of a relation matrix MD satisfy the following conditions:

(i) All diagonal entries are 1.

(ii) If v_i is infimum then, $a_{ii} = 1$, $\forall\ i$, and $a_{ij} = 0$ for $i \neq j$ where $i, j = 1, 2, ..., n$.

(iii) If v_i is a supremum then, $a_{ij} = 1$, $\forall\ j$. If at least one zero entry in a column for v_i that is $a_{ij} = 0$, for $\forall\ i$ then, v_i is a supremum.

(iv) The number of 1's in the corresponding row of a_{ij} entry gives the cardinality of a subarc connectivity $v_i v_j$.

Proof: Let, $\vec{G} = (V, X)$ be a multidisemigraph and let $(V, \Re)$ be a poset on $\vec{G}$. In the proof, for the sake of brevity we use the terms "adjacent" and "related" interchangeably. Let us consider the relations say $v_1 \Re v_k, v_2 \Re v_k, ..., v_{k-1} \Re v_k$, $1 \le k \le n$. That is there exists arcs $\vec{e_1}, \vec{e_2}, ..., \overrightarrow{e_{k-1}}$ in the multidisemigraph $\vec{G}$ or in other words v_1 is adjacent to v_k, v_2 is adjacent to v_k and continuing we end up with v_{k-1} is adjacent to v_k. But, we note that v_k is not related to v_i, $i = 1, 2, ..., k-1$. This implies that there exists an arc $\vec{e_k}$ connecting v_k and v_n. Now the entries in the relation matrix say, $MD = [a_{ij}]$ with respect to the relations $v_1 \Re v_k, v_2 \Re v_k, ..., v_{k-1} \Re v_k$ and are 1, whereas rest of the entries are 0.

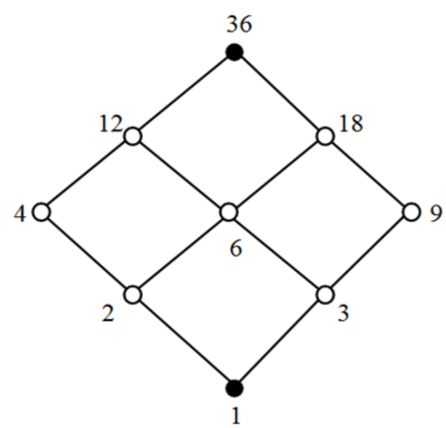

Fig. 3.8. Hasse Diagram of a relation $\Re$

Now, suppose $v_1 \Re v_n$ that is in a graphical sense if there exists a path $v_1 v_k v_n$ where v_1 and v_n are the end vertices of a subarc say $\vec{e_{i_1}}$, $1 \le i_1 \le n$ containing v_k as the middle vertex, then this connectivity implies that $v_1 \Re v_n$ through v_k. The entry corresponding to this relation is 1. By proceeding in this manner we can show that $v_2 \Re v_n$ through v_k and so on. Thus, we obtain a relation matrix MD corresponding to the multidisemigraph poset $(V, \Re)$ such that MD represents the multidisemigraph $\vec{G}$ uniquely. The above discussion implies that the conditions (i), (ii), (iii) and (iv) hold good for a multidisemigraph.

Conversely, (i) imply that $\Re$ is reflexive. Conditions (ii) and (iii) imply that $\Re$ is antisymmetric. If not, let $\exists\, a, b \in MD$ such that $a \Re b$ and $b \Re a$ where $a \neq b$ then obviously v_i is neither infimum nor supremum which is a contradiction to the conditions (ii) and (iii). Also, condition (iv) implies transitivity. If not, then the number of 1's in the corresponding row of infimum or supremum vertex will be different. With this it is obvious to conclude that relation of the matrix is MDSPOSET on V.

Theorem 3.23: The three dimensional rectangular co-ordinate system is an MDSPOSET with the subspace as a relation with $L=\left\{\{0,0,0\},\left\{X_{i_1},X_{i_2},X_{i_3}\right\},\left\{X_{i_1}X_{i_2},X_{i_2}X_{i_3},X_{i_3}X_{i_1}\right\},\left\{X_{i_1}X_{i_2}X_{i_3}\right\}\right\}$ as elements of the set $\mathfrak{R}_1$ with usual the meaning, X_{i_1},X_{i_2},X_{i_3} as co-ordinate axes, $X_{i_1}X_{i_2},X_{i_2}X_{i_3},X_{i_3}X_{i_1}$ as coordinate planes, X_{i_1},X_{i_2},X_{i_3} as three dimensional space $\mathfrak{R}^3$ and $\{0,0,0\}$ as the origin.

Proof: All the elements of L are vector spaces and we give the proof in different stages.

Stage 1: It is apparent that $\{(0,0,0)\}\subset\left\{X_{i_1}\right\}$ or $\left\{X_{i_2}\right\}$ or $\left\{X_{i_3}\right\}$

Stage 2: $X_{i_r}\subset\left\{X_{i_r}X_{i_s}\right\}$, $i_r\neq i_s$, $r\neq s$, $r=s=1,2,3$

Stage 3: $X_{i_1}X_{i_2}\subset X_{i_1}X_{i_2}X_{i_3}$, $i=1,2,3$

The above argument implies that

(i) $x\in L\Rightarrow x\mathfrak{R}\,x$
(ii) For $x,y\in L$ and $x\mathfrak{R}\,y$ implies y is not related to x
(iii) For $x,y,z\in L$ such that $x\mathfrak{R}\,y$ and $y\mathfrak{R}\,z$ implies that $x\mathfrak{R}z$.

The above discussion implies the following

(i) Every element of L is a subspace of itself. This proves reflexivity.
(ii) If A is a subspace of B then B cannot be a subspace of A , $\forall\, A,B\in L$ such that $A\neq B$. Otherwise, it contradicts the definition of proper subspace.
(iii) Obviously, $\forall\, A,B,C\in L$ and $A\subset B,B\subset C\Rightarrow A\subset C$

Therefore, L is an MDSPOSET subspace on the relation subspace.

The above theorem is illustrated by the following example.

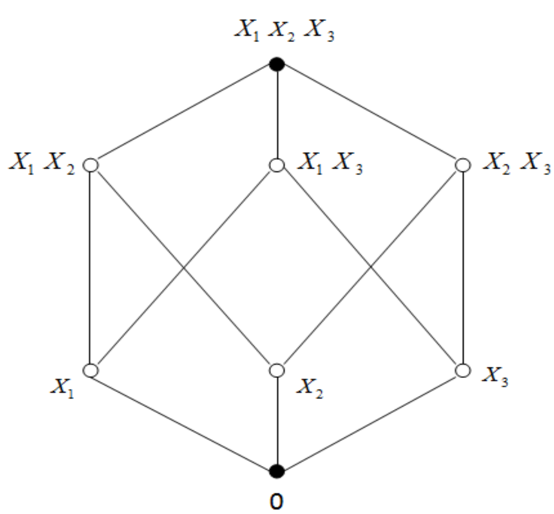

Fig. 3.9. Hasse diagram of $\mathfrak{R}^3$

Example 3.5: The MDSPOSET of a three dimensional rectangular co-ordinate system is represented by Hasse diagram. The co-ordinate $(0,0,0)$ which is the infimum, denoted by the end vertex and is related to X_1, X_2 and X_3. The co-ordinates X_1, X_2 and X_3 which are related to X_1X_2, X_2X_3 and X_1X_3 are denoted by the middle vertices. The co-ordinates X_1X_2, X_2X_3 and X_1X_3 denoted by the middle vertices are related to $X_1X_2X_3$. The element $X_1X_2X_3$ is the supremum and is denoted by the end vertex.

Lemma 3.24: The four dimensional orthogonal space $\Re^4 = (X_1X_2X_3X_4)$ is a vector space with the standard basis $\{(1\ 0\ 0\ 0)\ (0\ 1\ 0\ 0)\ (0\ 0\ 1\ 0)\ (0\ 0\ 0\ 1)\}$ and with the usual addition of the four tuples and scalar multiplication.

Proposition 3.25: The four dimensional orthogonal space $\Re^4$ of Lemma 3.24 induces an MDSPOSET for relation subspace on the set

$$L = \left\{ \begin{array}{l} (0\ 0\ 0\ 0), (\{X_1\}\{X_2\}\{X_3\}\{X_4\}), (\{X_1X_2\}\ \{X_1X_3\}\ \{X_1X_4\}\{X_2X_3\}\{X_2X_4\}\{X_3X_4\}), \\ (\{X_1X_2X_3\}\ \{X_1X_2X_4\}\{\{X_1X_3X_4\}\}\{X_2X_3X_4\}), (\{X_1X_2X_3X_4\}) \end{array} \right\}$$

Lemma 3.26: Let $\Re^n$ with X_i, $i = 1,2,.....,n$ be an orthogonal co-ordinate system. Then, $\Re^n$ is a vector space with the standard basis $\{a_1 = (1000----0), a_2 = (0100----0), ---, a_n = (000----1)\}$.

Here, $\Re^n$ denotes the collection of ordered $n-$tuples, n being a positive integer.

Theorem 3.27: The $n-$ dimensional $\Re^n$ space of Lemma 3.26, induces an MDSPOSET on $L = \{(0-----0), (X_{i_1}\ X_{i_2}........X_{i_k}) / 1 \le k \le n, X_{i_r} \ne X_{i_s} \forall\ r = s = 1,2,....,n\}$ with relation $\Re$ as subspace and with the usual addition of the four tuples and scalar multiplication.

Proof: We prove the theorem in stages, ranging from 0^{th} stage to $(k+1)^{th}$ stage.

0th Stage: Elements $\{(0.......0)\}$ $n-$tuple is vector space itself.

1st Stage: The elements of 1st stage are $\{X_{i_1}, X_{i_2}, ..., X_{i_n}\}$ and this subspace is related to X_{i_r}, for $r = 1,2,.....,n$.

2nd Stage: Elements are of the form $\{X_{i_r} X_{i_s} / r < s,\ r = s = 1,2,.....,n, r < s\}$. Also, $X_{i_r} \Re X_{i_r} X_{i_s}$, $\forall\ r = s = 1,2,....,n$.

3rd Stage: Elements are of the form $\{X_{i_r} X_{i_s} X_{i_t} /,\ r = s = t = 1,2,......,n,\ r < s < t\}$. Also, $X_{i_r} X_{i_s} \Re X_{i_r} X_{i_s} X_{i_t}$, $\forall\ r = s = 1,2,....,n$.

Proceeding up to $(k+1)^{th}$ stage, we get

$(k+1)^{th}$ **Stage:** Elements are of the form $\{X_{i_1} X_{i_2} .....X_k X_{i_{k+1}}\}$, where $i_1 = i_2 = ... = i_k = i_{k+1}$ with $i_1 < i_2 < ...... < i_k < i_{k+1}$ there exists ${}^nC_{k+1}$ element such that $X_{i_1} X_{i_2} .....X_{i_n} \Re\ X_{i_1} X_{i_2} ....X_k X_{i_{k+1}}$.

Continuing the argument we note that every k^{th} level element is related to at least $n-k$ elements of $(k+1)^{th}$ elements as are needless to prove that at least one k^{th} element is related to at least one element of $(k+1)^{th}$ level.

Thus, we summarize the above argument

(i) $x \in L \Rightarrow x \Re x$
(ii) For $x, y \in L$ and $x \Re y$ implies y is not related to x .
(iii) For $x, y, z \in L$ such that $x \Re y$ and $y \Re z$ implies that $x \Re z$.

Hence, $(L, \Re)$ is an MDSPOSET.

We note that the relaxation of the orthogonality condition in results 3.24, 3.25 and 3.26 generalizes the concept.

Corollary 3.28: $\Re^n$ MDSPOSET has the following properties

(i) Degree of each element is equivalent to outdegree.
(ii) One element with a degree n that is an element at 0^{th} level.
(iii) Every element at k^{th} level has a degree $n-k$.
(iv) The total number of vertices is ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$.
(v) The column corresponding to the k^{th} level has k 1's and $n-k$ 0's.
(vi) $id(v_i) + od(v_i) = n$.

4 Conclusion

a) In section 2, we state the basic terminology needed to read this paper.
b) In section 3, we introduce the concept of multidisemigraph poset, different types of relation such as reflexive relation, transitive relation, symmetric and antisymmetric relations.
c) In the Theorem 3.22, we prove that a multidisemigraph is a poset if and only if, it can be represented by the relation matrix MD .
d) In section 3.23, it is established that a three dimensional rectangular co-ordinate system is an MDSPOSET with subspace as the relation and is represented by Hasse diagram.
e) In section 3.26, it is established that the $n-$dimensional $\Re^n$ space induces an MDSPOSET on with relation $\Re$ as subspace.

Acknowledgements

The authors are thankful to the reviewers for their insightful comments on the paper, as these comments helped us to frame the paper in the required technical format.

Competing Interests

The authors declare that no competing interests exist.

References

[1] Sampathkumar E. Semigraphs and their application. Technical Report [DST/MS/022/94], Department of Science and Technology, Govt. of India, August, 1999.

[2] Kamath SS, Bhat RS. Domination in Semigraphs. Preprint submitted to Elsevier Science. 2003;106-111.

[3] Kamath SS, Hebbar Saroja. Domination critical semigraphs. Electronic Notes in Discrete Mathematics. 2003;15:113.

[4] Prabhakar R. Hampiholi, Jotiba P. Kitturkar. On enumeration of some semigraphs. British Journal of Mathematics and Computer Science. 2016;14(2):1-14.

[5] Hampiholi PR, Shirkol SS, Kaliwal MM. On the enumeration of path, unicyclic and bicyclic semigraphs with one or two middle vertices. Proceeding of National Conference on Recent Trends in Mathematics. 2016;1:28-34.

[6] Deshpande CM, Yogeshri Gaidhani, Athawale BP. Incidence matrix of a semigraph. British Journal of Mathematics and Computer Science. 2015:9(1):12-20.

[7] Frank Harary. Graph theory. Narosa Publishing House, New Delhi; 1988.

[8] Narsingh D. Graph theory with applications to engineering and computer science. New Delhi: Prentice Hall of India Private Limited; 2000.

[9] Sampathkumar E, Pushpalatha L. Matrix representation of semigraphs. Advanced Studies in contemporary Mathematics. 2007;14(1):103-109.

[10] Mathew Varkey TK, Susan RJ. On partially ordered disemigraphs. International Journal of Computation and Applied Mathematics. 2017;12(2); 503-509.

[11] Dushnik B, Miller E. Partially ordered sets. American Journal of Mathematics. The Johns Hapkins University Press. 1941;63(3):600-610.

© 2018 Hampiholi et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/24279