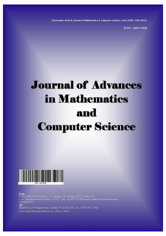

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Lepton Bound State Theory Based on First Principles

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

A quantum field theory has been constructed, in which leptons are bound by electromagnetic forces. Using severe boundary conditions, in particular several constraints on the rotation velocity, a precision test has been possible, in which the needed 7 parameters are determined by many more constraints. Since arbitrary adjustment parameters are excluded, absolute values of radii, rotation velocities and binding energies are obtained, possible only in a fundamental theory, which must be close to the *final* lepton theory. The resulting masses are obtained with uncertainties much smaller than 1 %.

The results show a very special structure of charged and neutral leptons.

1. Charged leptons: The deduced radii due to electric and magnetic binding are different by many orders of magnitude. In particular, the large electric root mean square radius of the electron of about 10^3 fm is almost of the same size as electron wave functions in light atoms, whereas the magnetic radius of $2.5 \cdot 10^{-10}$ fm is consistent with a "point" particle needed to describe electron-hadron scattering.

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2. Neutrals: The acceleration term gives rise to dynamically generated neutral particles of "hole" structure, which can be identified with neutrinos. Their masses are $2 \cdot 10^{-8}$ eV, 17 eV and 12 MeV for ν_e, ν_μ and ν_τ, respectively.

The full calculations together with the underlying fortran source code can be viewed at https://h2909473.stratoserver.net or https://leptonia-etc.de.

Keywords: Bound state formalism of leptons; all parameters constrained by a maximum number of conditions; very different electric and magnetic radii of charged leptons; neutral particles identified with neutrinos.

2010 Mathematics Subject Classification: 81T10, 46N50, 81P10, 83C47.

1 Introduction

The study of particle- and astrophysics is the only way to get information of the evolution of the universe. Since experimental facts on this subject are scarce, a correct picture can be obtained only, if the known conservation laws of physics are respected, relativity (coupling between space and time) is included in the formalism and astrophysical observations and particles properties are described accurately. In addition, we have to require that such a fundamental theory couples to the vacuum and is based on first principles.

Although efforts have been made in the past to construct such a theory, the sheer number of conditions appeared to be impossible to meet. Instead, effective field theories have been developed [1], combined in the Standard Model of particle physics (SM), see e.g. in ref. [2], which has been very successful in describing the wealth of data on electroweak interactions and the strong interaction in the perturbative regime at high energies. Because of its good performance, it could be argued that a fundamental theory should have a rather similar structure. This, however, cannot be the case, because in the SM a large number of more than 20 parameters are needed, which had to be adjusted to experimental data. These parameters comprise coupling parameters of three different fields with corresponding coupling constants, further 15 masses of basic particles, as leptons (concerned here), which are taken in the SM as point particles without inner structure. It is evident that the structure of these particles cannot be understood in this model. Also, serious obstacles are that the radial degree cannot be accessed and relativity and gravity cannot be included in this framework.

It should also be mentioned that extensions of the Standard Model have been constructed based on the very special SU(3)*SU(2)*U(1) symmetry structure of the SM, from which new particles have been predicted, as supersymmetric partners of known particles, axions, magnetic monopoles and other new particles. However, none of this additional particles could be detected experimentally, which attests our general concern that extensions of effective theories are questionable.

For the description of the universe gravitational theories have been discussed, mainly based on Einstein's theory of gravitation. These cosmological models describe the expansion of the universe, but show important shortcomings: Energy conservation is violated and the matter-antimatter non-equilibrium in the universe cannot be explained. Further, they require [3] the existence of dark matter (for which no evidence has been found in astrophysical studies and particle physics experiments) and dark energy, an unknown form of energy, which is not understood at all. Further, these classical models cannot be related to particle physics, for which a quantum description is needed.

Alternatively, efforts have been made to describe gravitation by string theories [4], which are extremely complex with hundreds of parameters and thus do not allow quantitative predictions.

What we really need is a basic theory of quantum structure, which can describe in a consistent way **all** properties of observed particles (also the radial properties) as well as the characteristics of gravity. Since massive particles can be considered as bound states of nature, such a theory should have a bound state structure. In addition, this description should take into account that nature develops from the highest degree of simplicity to more and more complex forms, which means that such a theory must have the most simple form possible. The consequences are that the corresponding Lagrangian must be as simple as possible and all wave functions, interactions, but also expectation values of radii and momenta should be of simple form. This requirement facilitates the construction of such a theory.

By generalising a successful bound state approach applied to hadron scattering from nuclei and hadronic systems [5], a theoretical framework has been developed, which satisfies all the above requirements [6]. It has a special (super-) symmetric structure of a fermion bound state, in which a second bound state of bosons is embedded. This particular structure leads to a dynamically generated confinement potential, which is needed in meson spectroscopy [7], and allows creation of particles out of the vacuum of fluctuating boson fields during overlap of bosons, see ref [8]. Further, this model generates the right fermion properties, which include static and dynamical terms. However, it is not clear, whether the boson bound state component is just needed to get the right binding of fermions, or allows also the existence of pure boson bound states, "glare-balls", in which fermions are missing (analog to "glueballs" discussed in quantum chromodynamics).

This bound state description has been applied to the description of atoms, hadrons and leptons [8, 9], but also to gravitation [6, 10]. Applied to the universe, this model [6] explains the genesis of the universe in a smooth way with creation of particles out of the vacuum[1], accumulation of a large mass, which became unstable and collapsed, leading to complete annihilation of antimatter. This gave rise to heating and expulsion of the surrounding matter to open space. In this picture also the accelerated expansion of the universe is explained [11]: at small radii gravitation is strong enough to reduce the expulsion of matter, whereas at large distances from the center gravitation is reduced, leading to an increased expansion. In this picture neither dark matter nor dark energy is needed.

Although this formalism allows an understanding of fundamental physics, further quantitative tests are needed. This is possible for systems, for which a maximum number of boundary conditions can be applied. Leptons are specially suited for such a test, because these particles are very stable and do not mix with hadrons of similar mass. In addition, the present model may allow to understand their intrinsic structure (not possible in the Standard Model, as mentioned above). Further, leptons exist also in the form of charge neutrals (neutrinos). The structure of these particles is also not known and their mass could not be measured. One can hope that the present analysis solves this problem.

Following the general formalism developed in ref. [6], some details are discussed, in particular on the generation of the radial degree of freedom, relativity and magnetic binding. Then, deduced wave functions, momenta and binding energies are discussed by applying a maximum set of boundary conditions.

2 Structure of the Underlying Bound States

For a fundamental theory we demand a Lagrangian of the most simple form. This is realized by a Lagrangian similar to that of quantum electrodynamics (QED), but with additional boson operators

$$\mathcal{L} = \frac{1}{\tilde{m}^2}(\bar{\Psi}D_\nu)\; i\gamma^\mu D_\mu\; (D^\nu\Psi) \;-\; \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\;, \tag{2.1}$$

[1] In this process only mass in form of binding energy is generated.

where $\tilde{m}$ is a mass parameter and Ψ massless charged fermion fields, $\Psi = \Psi^+$ and $\bar{\Psi} = \Psi^-$. Vector boson fields A_μ with coupling g to fermions are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$. The second term of the Lagrangian represents the Maxwell term with Abelian field strength tensors $F^{\mu\nu}$ given by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, which gives rise to both electric and magnetic coupling.

By inserting $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu\partial^\nu - ig(A_\nu\partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$, the Lagrangian $\mathcal{L}$ gives rise to a number of terms, which contain boson and fermion fields and/or their derivatives up to third order. From these terms two-body matrix elements can be derived, which can be written by $\mathcal{M} = \psi(p')\ K(q)\ \psi(p)$, where $\psi(p)$ are fermion wave functions $\psi(p) = \frac{1}{\tilde{m}^{3/2}}\Psi(p_1)\Psi(p_2)$ of scalar or vector structure (coupling of $\Psi(p_1)$ and $\Psi(p_2)$ to spin 0 or 1). Further, $K(q) = \frac{1}{\tilde{m}^{2(n+1)}}\gamma_\mu\gamma^\nu[O^n(q)O^n(q)]$, where $O^n(q)$ are cubic terms containing boson fields and/or derivatives obtained from the Lagrangian (2.1). These matrix elements are given explicitly in ref. [6]. The γ-matrices can be removed in the known way by adding a matrix element with interchanged indices.

It is very important to get access to the radial degree of freedom. To achieve this, a reduction of the fermion and boson fields by one dimension is needed, which requires that the 4-component p_4 of the fermion 4-momentum p is constant. For a bound state this is indeed the case, because p_4 can be related to the binding energy. This leads to energy-momentum conservation, one of the most important boundary conditions discussed below. Then, the momenta can be converted to radii by Fourier transformation.

Boson wave functions can be defined in a similar way by $W(q) = \frac{1}{\tilde{m}}A_\mu(q)A^\nu(q)$. Again, the momenta (for bosons only 3-momenta) can be reduced by one dimension, requiring that the three-component q_3 is constant (related to a boson binding energy). Fourier transformation leads then also to a conversion of the boson momenta to radii.

Concerning relativity, for a bound state as described above, there is only a simple space-time relation $t_{rot} = 2\pi < r^2 >^{1/2} /v_{rot}$, where t_{rot} and t_{rot} are the rotation time and velocity and $< r^2 >^{1/2}$ the average radius of the system. Since the radius and the rotation velocity can be calculated, relativity is established.

For magnetic binding the resulting binding energy components are given by

$$E_{ng} = 4\pi \int r^2 dr\ M_{ng}(r) = 4\pi \int r^2 dr\ \psi_{s,v}(r)\ V_{ng}(r)\ \psi_{s,v}(r)(v/c) \tag{2.2}$$

with potentials

$$V_{2g}(r) = \frac{\alpha^2(2s+1)(\hbar c)^2}{8\tilde{m}} \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big)\frac{(v/c)}{w_s(r)}\ , \tag{2.3}$$

where s=0 for scalar and s=1 for vector states, and

$$V_{3g}(r) = \frac{\alpha^2(\hbar c)}{\tilde{m}} \int dr'\ w_{s,v}(r')\ v_v(r-r')\ w_{s,v}(r')(v/c) \tag{2.4}$$

with fermion and boson wave functions $\psi_{s,v}(r)$ and $w_{s,v}(r)$ and an interaction $v_v(r) \sim -\alpha(\hbar c)\ w_v(r)$. It should be noted that in the spectroscopy of mesons [7] an empirical "confinement" potential with an almost linear rise to large radii was required to describe the levels of excited mesons. This potential can be identified with $V_{2g}(r)$, also responsible for the creation of particles out of the vacuum during overlap of boson fields [8].

The kinetic energies are given by

$$E^T_{ng} = \frac{4\pi}{2} \int r^3 dr\ T_{ng}(r) = \frac{4\pi}{2} \int r^3 dr\ \psi_{s,v}(r)\ V^T_{ng}(r)\ \frac{d\psi_{s,v}(r)}{dr}(v/c) \tag{2.5}$$

with

$$V^T_{1g}(r) = \frac{\alpha(2s+1)(\hbar c)^3}{4\tilde{m}^2} \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big)(v/c) \tag{2.6}$$

and

$$V^T_{2g}(r) = \frac{2\alpha^2(\hbar c)^2}{\tilde{m}}\ w_{s,v}(r) w_{s,v}(r)(v/c)\ . \tag{2.7}$$

Finally, there is an acceleration term

$$\Delta E_{1g} = \frac{4\pi}{2}\int r^4 dr\ B_{1g}(r) = \frac{4\pi}{2}\alpha(\hbar c)\int r^4 dr\ \psi_{s,v}(r)\ w_{s,v}(r)\ \frac{d^2\psi_{s,v}(r)}{dr^2}(v/c)^2\ . \tag{2.8}$$

For this term the density $\psi_{s,v}(r)\frac{d^2\psi_{s,v}(r)}{dr^2}$ cancels out and cannot contribute to the binding energy. Therefore, the total (fermion) mass is given by $M^f = -(\sum_n E_n + E^T_n)$.

For bosons the corresponding binding energy scales with the number N of bosons

$$E^g = 2\pi\int rdr\ M^g(r) = 2\pi N\alpha^2\int rdr\ w_{s,v}(r)\ v_v(r)\ w_{s,v}(r)(v/c) \tag{2.9}$$

whereas the kinetic energy is given by

$$E^g_T = \frac{2\pi}{2}\int r^2 dr\ T^g(r) = 2\pi N\frac{\alpha^2(\hbar c)}{4}\int r^2 dr\ w_s(r)\ \frac{1}{r}\frac{dw_s(r)}{dr}(v/c)\ . \tag{2.10}$$

Finally there is also a contribution from acceleration

$$\Delta E^g = \frac{2\pi}{2}\int r^2 dr\ B^g(r) = 2\pi N\frac{\alpha^2(\hbar c)}{8}\int r^2 dr\ w_s(r)\ \frac{d^2 w_s(r)}{dr^2}(v/c)\ . \tag{2.11}$$

For leptons of $(q\bar{q})q$ or $(q\bar{q})\bar{q}$ structure (q being massless elementary fermions) we can assume also a three boson structure ($N = 3$). The total boson binding energy is given by $E^g_{tot} = (E^g + E^T_g + \Delta E^g)$.

To summarize the most important features: The third order Lagrangian (2.1) gives rise to a complete bound state description of particles with potential and kinetic energy, but also with acceleration terms, specially important for the evolution of complex systems [6]. All elementary fermions are massless, giving rise to a coupling to the vacuum. By generating the radial degree of freedom by Fourier transformation of the corresponding momenta and extracting the rotation time from the corresponding velocity, relativity (the coupling of space and time) is realized. To respect the uncertainty principle, particle positions are used in the form of wave functions (or densities). Further, severe boundary conditions can be defined, by which all relevant parameters of the model can be determined. This leads to a description of particles solely bound by electromagnetic interactions.

2.1 Boundary conditions

To be able to evaluate these energy components, the wave functions $\psi(r)$ and $w(r)$ have to be determined, further the rotation velocity (v/c) and the coupling constant α. The wave functions are fixed entirely by simple geometric constraints

$$\psi_s(r) \sim w_s(r) \tag{2.12}$$

with a normalization $4\pi\int r^2 dr\ \psi^2(r) = 1$ for fermions and $2\pi\int rdr\ w^2(r) = 1$ for bosons. A second constraint connects the potential $V_{3g}(r)$ to the boson wave function

$$|V^v_{3g}(r)| \sim w^2_s(r)\ . \tag{2.13}$$

This condition requires simply that the interaction potentials cannot act at larger radii than the density $w^2_s(r)$. Both constraints are fulfilled by a form of the scalar wave function

$$w_s(r) = w_{s_o}\ exp\{-(r/b)^\kappa\}\ , \tag{2.14}$$

whereas the orthogonal wave function of vector structure is given by

$$w_v(r) \sim [w_s(r) + \beta R \; \frac{dw_s(r)}{dr}] \; , \tag{2.15}$$

where $\beta R = -\int r^2 dr \; w_s(r) / \int r^2 dr \; \frac{dw_s(r)}{dr}$.

Important to note, because of the different normalization of the fermion and boson wave functions this form of wave functions should be valid for any basic system of two or more elementary particles.

From the potentials (2.3) or (2.4) a mass-radius constraint can be derived

$$Rat_{2g} = \frac{(\hbar c)^2 (v/c)^2}{\tilde{m}^2 < r^2_{w_s} >} = 1 \; , \tag{2.16}$$

where $< r^2_{w_s} >$ is the mean radius square of the boson wave function $w_s(r)$. This radius can be related to the average rotation time of the system, giving rise to a space-time dependence required from relativity.

In addition, for particles in the vacuum there is conservation of linear momentum

$$< q_f^2 >^{1/2} + < q_g^2 >^{1/2} = 0 \; , \tag{2.17}$$

where $< q_f^2 >^{1/2}$ and $< q_g^2 >^{1/2}$ are the root mean square momenta of the total fermion and boson potentials. This condition gives rise to recoil corrections for bosons, which are rather small. To determine these momenta, the corresponding density and potential functions $P_{f,g}(r)$ are Fourier transformed, $P_f(q) = 4\pi(\hbar c) \int r^2 dr \; sin(qr)/qr \; P_f(r)$ for fermions and $P_g(q) = 2\pi(\hbar c) \int rdr \; sin(qr)/qr \; P_g(r)$ for bosons.

An essential condition is related to the reduction of the fermion and boson momenta by one dimension, as discussed above. This leads to energy-momentum conservation

$$< q^2_{f,g} >^{1/2} (v/c) = -E_{f,g} \; . \tag{2.18}$$

Combined with relation (2.17) this requires also similar fermion and boson energies $E_f = E_g$. The condition (2.18) is also imperative to establish full relativity, since only by this condition Fourier transformation of the momenta to the radial degree of freedom is allowed and can connect to the time of rotation.

The last two conditions connect the rotation velocity to the total kinetic energy of fermions and bosons

$$(v/c)_f^{tot} = 2 \cdot 2 \sqrt{\frac{-(E^T_{1g} + E^T_{2g})}{2\tilde{m}}} (v/c) \tag{2.19}$$

and

$$(v/c)_g^{tot} = 2 \sqrt{\frac{-(E^g_T + f\Delta E^g)}{2\tilde{m}}} (v/c) \; . \tag{2.20}$$

The additional factor 2 in eq. (2.19) is due to the fact that the kinetic energies E^T_{1g} and E^T_{2g} are deduced from the interaction with one particle pair, whereas the total kinetic energy is given by the contribution from all three fermions. For bosons a significant fraction of the acceleration term $f\Delta E^g$ contributes to the rotation velocity (2.20), and f is determined by adjusting eq. (2.20) to the fitted velocity (v/c), which should be the same for all four relations (2.16), (2.18) - (2.20).

3 Structure of Charged Leptons

Coming to the calculations of leptons, in a first analysis assuming magnetic binding only [9], the dynamics has been taken from a Hamilton approximation, replacing simply $V^T(r)$ by the derivative of $V(r)$. Differently, in the present analysis the full dynamics has been taken into account explicitly with all terms specified above.

In the above formalism electric and magnetic binding is described within the same formalism (using $(v/c) = 1$ in case of electric binding). For electric binding we have therefore only three parameters for each lepton system, the slope and shape parameters b and κ, see eq. (2.14), and the coupling constant α, whereas the description of magnetic binding requires adjustment of (v/c) as well. This gives in total seven parameters for each lepton system. One other small adjustment was needed to get full consistency: only for electrically bound electrons E^T_{2g} had to be increased by a factor of 1.2. But this adjustment is needed to get a binding energy in full agreement with all other constraints.

If we count the number of constraints, for electric and magnetic binding there are two geometric conditions on the shape of the wave functions, two bounds on linear momentum, two on energy-momentum conservation, two mass-radius conditions and five relations on the rotation velocity (v/c), in total more than 20 conditions. Further constraints arise, if one requires that the bound state structure of all leptons is the same with one coupling constant α. This shows that not only all model parameters are rigorously determined, in addition also a detailed check of the formalism is possible. Only if all this matches precisely, a quantitative description becomes possible. Such hard conditions could never be applied to any particle description and it possible only in a really basic and fundamental theory.

Table 1. Parameters b and (v/c) (for $\kappa = 1.375$ and $\alpha = 2.158$) and resulting fermion radii and momenta, momenta multiplied with (v/c), and (fermion) masses for electric and magnetic binding. All dimensions are in fm or MeV, respectively

System	b	(v/c)	$< r_f^2 >^{1/2}$	$< q_f^2 >^{1/2}$	$< q_f^2 >^{1/2}$ (v/c)	mass$_f$
τ elec	$2.547\ 10^{-1}$	1	0.290	1757	1757	1777
τ mag	$9.60\ 10^{-3}$	$3.77\ 10^{-2}$	$1.093\ 10^{-2}$	46.60	1757	1777
μ elec	4.2834	1	4.876	104.5	104.5	105.7
μ mag	$3.90\ 10^{-6}$	$9.11\ 10^{-7}$	$4.44\ 10^{-6}$	$1.15\ 10^{5}$	104.5	105.7
e elec	$8.845\ 10^{2}$	1	$1.007\ 10^{3}$	0.5053	0.5053	0.512
e mag	$2.10\ 10^{-10}$	$2.37\ 10^{-13}$	$2.461\ 10^{-10}$	$2.131\ 10^{9}$	0.5052	0.511

Table 2. Fitted radial velocity (v/c) compared to that deduced from the mass-radius constraint (2.16) and the total kinetic energies $E^{tot}_{kin} = (\frac{Mv}{2c})^2$ of fermions and bosons, see eqs. (2.19) and (2.20), using $f = 0.71$

system	$(v/c)_{fit}$	mass-rad	$E^{tot}_{kin}(ferm)$	$E^{tot}_{kin}(bos)$
τ elec	1	1.000	1.001	1.000
τ mag	$3.770\ 10^{-2}$	$3.770\ 10^{-2}$	$3.770\ 10^{-2}$	$3.770\ 10^{-2}$
μ elec	1	1.000	1.000	1.000
μ mag	$9.106\ 10^{-7}$	$9.106\ 10^{-7}$	$9.108\ 10^{-7}$	$9.103\ 10^{-7}$
e elec	1	0.999	1.00*	1.000
e mag	$2.371\ 10^{-13}$	$2.371\ 10^{-13}$	$0.2371\ 10^{-13}$	$2.370\ 10^{-13}$

* *E^T_{2g} increased by factor 1.2*

Here we concentrate on the discussion of lepton s-states (two fermions in a relative s-state). Starting with the parameters $\kappa = 1.35$ and $\alpha = 2.14$ from previous analyses, we found that small adjustments are needed. With slightly larger values, $\kappa = 1.375$ and $\alpha = 2.158$, electric and magnetic binding of all three systems are well described. Then b is the only parameter to be adjusted for electric binding. For magnetic binding b and (v/c) had to be adjusted, with the best value of b consistent with that deduced in ref. [9]. In table 1 the fitted values of b and (v/c) are given for all three systems in the first two rows, then follows the root mean square radius of the fermion wave function in the 3^{rd} row. Resulting momenta $< q_f^2 >^{1/2}$, $< q_f^2 >^{1/2}$ (v/c) and final masses are given in the next rows. In the last row one can see that the masses are well described within errors much smaller than 1 %.

In the determination of the final parameters the rotation velocity (v/c) is very important, it enters in all expressions of binding energies, but also in the mass-radius constraint (2.16). Therefore, the additional constraints (2.19) and (2.20) are essential for a precise parameter determination: the uncertainties in κ and α are strongly reduced with respect to previous analyses, and small ambiguities between b and (v/c) are eliminated; further, the deduced (fermion) masses are very close to the experimental values. The resulting rotation velocities (v/c) extracted from the different conditions are given in table 2. They show very small differences, indicating that a very consistent description in the present formalism is achieved.

Concerning the momenta in relation (2.17), the boson momentum $< q_g^2 >^{1/2}$ is only about 2 % smaller than $< q_f^2 >^{1/2}$ for all three lepton systems. This shows that previous analyses were quite reasonable, in which recoil effects have not been corrected. Taking a recoil correction $1+(< q_f^2 >^{1/2} - < q_g^2 >^{1/2})/ < q_g^2 >^{1/2}$ on the binding of bosons into account, very similar binding energies of fermions E^f and boson E^g are obtained, as seen in table 3. Still, with a better fine tuning of the parameters the small binding energy differences between fermions and boson binding of about 0.1 % might disappear completely. This shows also that the numerical uncertainties in the Fourier transformations and integration limits due to radial and momentum cut-offs are very small.

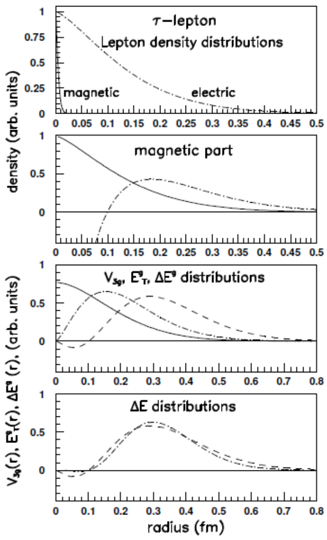

Fig. 1. Radial dependence of the densities of the τ-lepton system. Upper part: Densities of electric (dot-dashed line) and magnetic binding (solid line). Upper middle part: Density of magnetic binding as in the upper part (solid line) in comparison with that of the acceleration term $\Delta E_2^g(r)$ (dot-dashed line). Lower middle part: τ-binding potential $V_{3g}(r)$ (solid line) in comparison with the kinetic energy distribution $E_T^g(r)$ (dot-dashed) and $\Delta E_2^g(r)$ (dashed line). Lower part: Acceleration terms $\Delta E^f(r)$ and $\Delta E_2^g(r)$, given by dot-dashed and dashed lines, respectively, representing neutral particle (τ-neutrino) energy distributions

Table 3. Binding energies in MeV for the different lepton systems taking recoil corrections $rec = 1 + (< q_f^2 >^{1/2} - < q_g^2 >^{1/2})/ < q_g^2 >^{1/2}$ for bosons into account. The last column shows the neutrino binding energies ΔE^f and ΔE_2^g

system	part.	E_{pot}	E_{kin}	ΔE^f	E_{tot}	$\Delta E(\nu)$
τ elec	fermions	-949.1	-827.8	–	-1777	
	bosons	-802.1	-697.7	-278.0	-1778	
τ mag	fermions	-949.3	-828.2	–	-1777	-11.61
	bosons	-802.2	-697.7	-278.0	-1778	-11.31
μ elec	fermions	-56.44	-49.22	–	-105.7	
	bosons	-47.69	-41.48	-16.53	-105.7	
μ mag	fermions	-56.45	-49.24	–	-105.7	-16.68 10^{-6}
	bosons	-47.70	-41.49	-16.53	-105.7	-16.32 10^{-6}
e elec	fermions	-0.294	-0.218*	–	-0.512	
	bosons	-0.231	-0.201	-0.080	-0.512	
e mag	fermions	-0.273	-0.238	–	-0.511	-2.10 10^{-14}
	bosons	-0.231	-0.201	-0.080	-0.511	-2.10 10^{-14}

** E_{2g}^T increased by factor 1.2*

An interesting property of the present theory is that the radii of electric and magnetic binding need to be very different, because $(v/c) = 1$ for electric and $(v/c) < 1$ for magnetic binding. This is seen in the average radii in table 1. Further, electric and magnetic binding had to be tuned separately to give the total binding energies, showing that there is no interference between these different bindings. The fact that the velocities and densities of electric and magnetic binding are not disturbed by each other confirms the assumption that elementary fermions and bosons need to be massless.

The densities of electric and magnetic binding of the τ-lepton are shown in the upper part of fig. 1, with a radial extent different by orders of magnitude. For the lighter systems these differences are even larger. For the electron this result is in full agreement with the experimental information: The large radius of the electric part is right between the proton radius of about 1 fm and atomic radii in the order of 100 pm, see e.g. ref. [8]. Differently, in the analysis of electron-hadron scattering the electron had to be considered as "point" particle [12]. This is consistent with the present result of an extremely small magnetic radius of about $2.5 \cdot 10^{-10}$ fm.

In the second part of fig. 1 the radial dependence of the scalar density $\psi_s^2(r)$ of the τ-lepton is compared to that of the acceleration term $\psi_s(r)\ d^2\psi(r)/dr^2$. Both densities are orthogonal to each other $\int \psi_s^2(r)\ \psi_s(r)\ d^2\psi(_sr)/dr^2 = 0$. Below one finds a comparison of the potential $V_{3g}^s(r)$ with the kinetic energy term $E_{2g}^T(r)$ and the acceleration term $\Delta E_{1g}(r)$, which are both shifted to larger radii. The mean radii of the fermion and boson acceleration terms $\Delta E_{1g}^T(r)$ and $\Delta E^g(r)$ given in the lower part are larger than that of the potential $V_{3g}^s(r)$ by roughly a factor of two.

4 Dynamical Structure of Neutrinos

In addition to the charged leptons $l^{\pm}_{e,\mu,\tau} = (e, \mu, \tau)^{\pm}$ there are neutrinos $\nu_{e,\mu,\tau}$ and $\bar{\nu}_{e,\mu,\tau}$, which are directly related to the corresponding leptons, $\nu_l \rightarrow l^+$ and $\bar{\nu}_l \rightarrow l^-$. The structure of these particles is up to now completely unknown; further their mass is relatively small and could not be measured. Since neutrinos appear only in relation to charged leptons, they should be generated in a valid lepton theory.

For a neutral particle the integration of the charge density over full space must cancel out: $4\pi Q \int r^2dr\ \psi^2(r) = 0$ (where Q is the electric charge). This is the case only for the acceleration term ΔE^f, which does not contribute to the fermion binding energy of charged leptons. This term scales with $(v/c)^2$. Differently, the corresponding boson term ΔE^g participates to the boson binding energy and scales with (v/c).

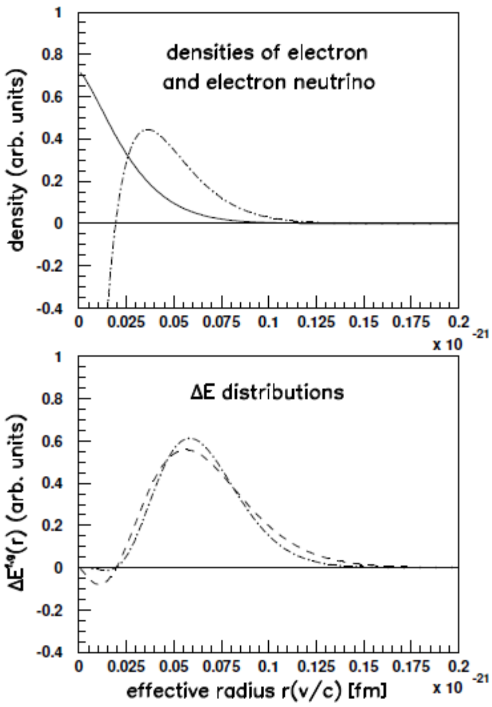

Fig. 2. Radial dependence of the densities similar to fig. 1 for the electron system (upper part) and the distributions $\Delta E^f(r)$ and $\Delta E_2^g(r)$ (lower part) of the electron neutrino

Assuming for bosons a second acceleration term with the same structure as eq. (2.11) but with a rotation velocity $(v/c)^2$

$$\Delta E_2^g = 2\pi N \frac{\alpha^2(\hbar c)}{8} \int r^2 dr\ w_s(r)\ \frac{d^2 w_s(r)}{dr^2}(v/c)^2 \tag{4.1}$$

leads to a rather similar fermion and boson structure, as shown in the lower parts of figs. 1 and 2 for the tau and electron system. For the τ-system this yields mean radii $< r_{f,g}^2 >^{1/2}$ (v/c) of 5.94 10^{-4} fm and 5.81 10^{-4} fm, respectively, which are significantly larger than the corresponding values of the total system. After Fourier transformation and proper recoil correction the value of ΔE_2^g is close to ΔE^f for all three lepton systems, see table 3. This points to the existence of a neutral particle with a characteristic "hole" structure (depression in the center), which can be identified as neutrino.

The acceleration terms contain only wave functions, but not the interaction $v_v(r)$, indicating that the neutrino is not bound to the charged lepton. Consequently, these particles can be considered as separate particles with the same rotation $(v/c)^2$ for fermions and bosons. This is further supported by the fact that the recoil correction $rec(\nu) \sim 1.22$ is significantly different from that for charged leptons ($rec \sim 1.02$). Remarkably, only half of the calculated recoil correction $rec(\nu)' = 1 + 1/2(< q^2_{\nu f} >^{1/2} - < q^2_{\nu g} >^{1/2})/ < q^2_{\nu g} >^{1/2}$ is needed to get agreement between ΔE^f and ΔE^g_2, indicating that the neutrino has also a kinetic energy, as expected for a bound state of nature. Indeed, if one writes $E^{\nu}_{kin} = \frac{M_\nu}{2}(v/c)^2$, where $M_\nu = -\Delta E(\nu)$, one finds that half of the binding energy (or mass) is due to rotation $E^{\nu}_{kin} = M_\nu/2$. This yields $\Delta E^f \simeq \Delta E^g_2$ with errors of ≤ 0.5 %.

Comparing the present results with experimental information, the neutrino masses are consistent with previous estimates, $M(\nu_e) < 2$ eV, $M(\nu_\mu) < 0.19$ MeV and $M(\nu_\tau) < 18.2$ MeV, see [2], but also with the much more restrictive mass limit $\sum M_{\nu_i} < 28$ eV from astrophysical observations [13]. The much heavier τ-neutrinos accompanying τ-leptons can be observed only in high energy experiments.

Since the neutrino binding energies in table 3 are different by many orders of magnitude, our results disagree with the picture of "neutrino oscillations", in which one assumes that neutrinos can change from one species to another, $\nu_a \rightarrow \nu_b$, where ν_a and ν_b are one of the three neutrinos. This could be possible only if the neutrino masses would be quite similar. This picture has been introduced to solve the "Solar neutrino" problem, the fact that the number of detected electron neutrinos from the sun is much less than predicted. However, due to the extremely small ν_e mass a large fraction of these neutrinos might be absorbed already in the sun and thus do not arrive at terrestrial detectors.

Although the above explanation of neutrinos is the only one possible in the present lepton theory, one has to discuss, whether neutrinos might not be due to a completely different mechanism, as an interference between two charge states of hadrons. Thinking of the decay of neutrons $n \rightarrow p\ e\ \bar{\nu}_e$, neutrinos could be related to the structure of p or n. However, the fact that the neutron has no charge may already indicate that it has an interference structure between two charged baryon states. Therefore, it seems to be unlikely that in this process two neutral particles appear, neutron and neutrino. A much stronger argument against such an interpretation is that in this picture the muon neutrino should be related to Σ and Λ (or another charged-neutral baryon doublet BD) and ν_τ to still another heavier baryon doublet BD', but a relation of such baryon doublets to neutrinos $BD \rightarrow (\mu, \nu_\mu)$ and $BD' \rightarrow (\tau, \nu_\tau)$ could never be found (only the coupling of $\nu_l \rightarrow l$ is established). This makes such a picture unrealistic.

5 Summary

In the present bound state theory a quantitative description of leptons has been obtained, in which not only the model parameters are well determined, moreover a strong reduction of the uncertainties in the results is achieved by using a maximum set of boundary conditions. Since arbitrary adjustment parameters are excluded, absolute values of radii, rotation velocities and binding energies are obtained, possible only in a fundamental theory, which is definitively close to the final lepton theory. Only marginal improvement might be possible by finding better criteria for the extraction of the fermion and boson wave functions.

The further results are:

1. Electric and magnetic binding gives rise to strongly different lepton radii. For the electron this explains an electric structure with radii in the picometer range, on the other hand the "point" structure needed in electron-hadron scattering.

2. The present theory provides a first consistent explanation of the nature of neutrinos. These particles are due to the acceleration terms and are separate systems decoupled from their charged partners. They satisfy all requirement of bound states of nature, leading to binding energies again with uncertainties of < 1 %.

Details of the calculations of radii, momenta, binding energies and rotation velocities, as well as the underlying fortran source code can be viewed at https://leptonia-etc.de or https://h2909473.stratoserver.net. Some details of the on-line calculations are given in the appendix.

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Competing Interests

The author declares that no competing interests exist.

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Appendix: On-line calculations of the different lepton systems

Step 1: Open up webside https://h2909473.stratoserver.net or https://leptonia-etc.de.

Step 2: The button "LEPTONS e μ τ" leads to the lepton page, where one can select by radio buttons calculations for electron, muon or tau. Also in the table above the used parameters are shown, on the left for electric binding, on the right for magnetic binding. The calculation starts by clicking on the button "calculate case".

Step 3: The fortran program starts in *main* by confirming the selected case. With parameter isch=1 it calculates electric binding and writes the corresponding parameters into the parameter file fune(5).

Step 4: In subroutine *para* the arrays of radii R(I) and momenta q(I) are calculated and links are made to the different subroutines *density, folding, confine, dopfold, virial* and *virialg*.

Step 5: In subroutine *density* the wave functions and densities are calculated and mean root square radii are shown for bosons and fermions in the first line of the output page.

Step 6: In subroutine *folding* the folding potentials $V_{3g}(r)$ and the dynamical terms are calculated with the corresponding radii given in the second line.

Step 7: The subroutine *confine* calculates the confinement potential $V_{2g}(r)$.

Step 8: In subroutine *dopfold* the fermion momentum distributions are calculated with root mean square momentum given in the output line 4. The values of energy-momentum conservation for bosons are given in output line 3, for fermions in line 5. The difference between these numbers indicated that a recoil correction for bosons is needed with an amplitude given in line 6.

Step 9: The next four output lines are generated by the subroutine *virial*, which calculates all components to the binding energy from the potentials $V_{3g}(r)$ and $V_{2g}(r)$ and the dynamic terms calculated in subroutine *folding*. Below the fitted rotation velocities (v/c) are given, calculated from different relations discussed in the text.

Step 10: Then the corresponding binding energies and velocities (v/c) are calculated by the subroutine *virialg* and displayed under "boson binding energies".

Step 11: By change of the parameter isch to 2 all calculations start from the beginning with the parameters for magnetic binding. Everything rests the same, only the binding energies of a neutral particle (neutrino) are shown in addition, highlighted by pink color.

A full calculation compiled with gnu-fortran *gfortran -o lept_fortran.cgi lept_fortran.cgi.f* and run with *lept_fortran.cgi* on a Linux operating system takes only part of a second. The program has been developed during many years; therefore it has not an effective structure and contains many additional options, which are not used for this particular presentation.

To get access to the fortran source code - which is exactly the same for all three different leptons cases - one clicks on the button "view source code".

If in addition an interested reader wants to check the calculations by himself, he can download the source code, compile it with gfortran or another version of fortran and run it. Even by varying the parameters he can try to find other solutions, which could belong to new bound states.

Final comment: Within the same formalism studies have been made for completely different systems, mesons and vector bosons, see ref. [14]. With the present fortran program again a quantitative description of these systems is achieved. These calculations can be run by chosing the button "MESONS and HEAVY BOSONS" on the start page and following similar instructions as for leptons. A third option "VECTOR STATES" exists on the start page, which allows to run calculations for vector states related to the mesons and heavy bosons discussed in ref. [14]. However, for these states of much larger masses changes in the boson part are needed, which shall be discussed in a future publication.

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