



# A Fractional Discrete Grey Model with Particle Swarm Optimizer and Its Applications in Forecasting the Gasoline Consumption in Chongqing China

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## Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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## Abstract

Forecasting gasoline consumption is of great significance for formulating oil production, foreign trade policies, and ensuring the balance of domestic refined oil supply. Based on grey system theory, a fractional accumulation operator is constructed to optimize the accumulation method of the traditional discrete grey model, and the Particle Swarm Optimization algorithm is used to solve the fractional nonlinear parameters. This model was used in the prediction of gasoline consumption in Chongqing, China, and compared with the existing 7 models. The results show that the fractional discrete grey model optimized by PSO has better prediction accuracy. The fractional discrete grey model optimized by PSO can be used as a quantitative method in the field of energy forecasting.

Keywords: Grey system; Fractional order accumulation; Particle swarm optimization; Gasoline consumption.

## 1 Introduction

With the rapid economic development, energy consumption is also increasing. China's oil consumption has maintained a relatively high growth rate. In 2018, China's energy consumption reached 471,925 (million tons of

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standard coal), and oil accounted for 18.9% of total energy consumption. The consumption of crude oil reached 63,004,33 (ten thousand tons), of which the consumption of diesel reached 164.0956 (ten thousand tons), and the consumption of gasoline reached 13,055.30 (ten thousand tons). In this context, predicting the future consumption of gasoline in my country is of great significance for formulating petroleum production, foreign trade policies, and ensuring the balance of domestic refined oil supply.

Grey system theory is a method based on "small data, poor information" uncertain system established by the famous Chinese scholar Professor Deng Julong in 1982 [1]. With the continuous development of grey system theory, grey model has become one of the most active branches in grey system theory. Scholars have conducted in-depth research on grey model from different angles. Chen Junzhen [2] and Tien [3] point out that the traditional grey model does not satisfy the principle of minimum information, so the initial value of the grey model is optimized; Tan Guanjun [4] first propose the background value concept of the grey GM(1,1) model, It reveals the root cause of the error source of the traditional grey model; Professor Deng Julong [5] believes that cumulative generation can change the grey process from grey to white, and through the accumulation, the law contained in the disordered original data is fully revealed, and then the development trend of the accumulation of ash can be seen. Song Zhongmin [6] conducts in-depth research on sequence accumulation generation, analyzes the nature of accumulation generation sequence, and proposes accumulation generation space and reverse accumulation generation operator; Qian Wuyong [7] etc., Wei Yuming [8] etc., Both Sun Quanmin and Wang Yapeng [9] point out that the accumulation sequence is the weighted sum of the original data. The traditional accumulation treats the original data as equally important, but it does not reflect the importance of the new information in the original data. At present, the grey forecast model has been widely used in transportation [10], urban water volume [11], power load [12], etc.

Intelligent optimization algorithms are inspired by human intelligence, the social nature of biological groups, or the laws of natural phenomena. People have invented many intelligent optimization algorithms, mainly including: Genetic algorithm [13], Ant Colony algorithm [14], Particle Swarm Optimization algorithm [15], etc. The swarm intelligence optimization algorithm mainly simulates the swarm behavior of insects, animal swarms, birds swarms and fish swarms. These swarms search for food in a cooperative manner. Each member of the swarm learns its own experience and the experience of other members. Constantly change the direction of the search. The outstanding feature of the swarm intelligence optimization algorithm is to use the swarm intelligence of the population to perform collaborative search to find the optimal solution in the solution space. Among them, the particle swarm optimization algorithm was proposed by Dr. Eberhart and Dr. Kennedy in 1995. It originated from the study of bird predation behavior. And it is widely used in agriculture [16], aviation [17] and electric power [21].

It can be seen from the above literature that optimizing the initial value of the grey model can effectively improve the prediction accuracy of the model. Therefore, this paper defines a fractional accumulation operator on the basis of the discrete grey model to improve its background value, and uses the particle swarm optimization algorithm to solve the optimal fractional parameters to further improve the stability and prediction accuracy of the model.

## 2 Material and Methods

### 2.1 Traditional discrete grey model

**Definition 1:** Let the sequence  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(m))$ ,  $X^{(1)}$  is the first-order cumulative generating sequence of  $X^{(0)}$ , namely

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(m)) \quad (1)$$

Where

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, m \quad (2)$$

Then called

$$x^{(1)}(k + 1) = \beta_1 x^{(1)}(k) + \beta_2 \tag{3}$$

is the DGM(1,1) model.

**Theorem 1:** The recursive solution of the DGM(1,1) model is

$$\hat{x}^{(1)}(k + 1) = \beta_1^k x^{(0)}(1) + \frac{1 - \beta_1^k}{\beta_1} \beta_2 \tag{4}$$

or

$$\hat{x}^{(1)}(k + 1) = \beta_1^k (x^{(0)}(1) - \frac{\beta_2}{1 - \beta_1}) + \frac{\beta_2}{1 - \beta_1} \tag{5}$$

The reduction formula is

$$x^{(0)}(k + 1) = x^{(1)}(k + 1) - x^{(1)}(k), k = 1, 2 \dots m \tag{6}$$

**Theorem 2:** Set  $X^{(0)}, X^{(1)}$  as shown in definition 1, then the parameter  $\hat{a} = [\beta_1, \beta_2]^T$  of the DGM(1,1) model is the least square estimate Satisfy:

$$\hat{a} = B^T (B^T B)^{-1} Y \tag{7}$$

Where

$$B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(m - 1) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(m) \end{bmatrix} \tag{8}$$

## 2.2 Gasoline consumption forecast based on discrete grey model of fractional accumulator optimized by PSO algorithm

Stability is a problem that must be considered in the study of any system. In the case of equal disturbances in the grey first-order accumulation method, the newer the data is disturbed, the smaller the disturbance bound of the solution, and the newer the data, the smaller the impact on the solution. This contradicts the principle of new information priority. When the order is less than 1, this contradiction is alleviated. Therefore, this paper uses the fractional cumulative grey model to enhance the stability of the grey model solution.

### 2.2.1 Model building

**Definition 2:** Let the non-negative sequence  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(m))$ , Said

$$x^{(r)}(k) = \sum_{i=1}^k C_{k-i+r-1}^{k-i} x^{(0)}(i) \tag{9}$$

is the accumulative operator of order  $r(0 < r < 1)$ , specifying  $C_{r-1}^0 = 1, C_k^{k+1} = 0, k = 0, 1, \dots, m - 1$ ,

$$C_{k-i+r-1}^{k-i} = \frac{(k - i + r - 1)(k - i + r - 2) \dots (r + 1)r}{(k - i)!} \tag{10}$$

Call  $X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(m))$  as  $r(0 < r < 1)$ -order cumulative sequence.

**Definition 3:** Let the non-negative sequence  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(m))$ , Then said

$$\alpha^{(1)}x^{(1-r)}(k) = x^{(1-r)}(k) - x^{(1-r)}(k - 1) \tag{11}$$

is an accumulative subtraction operator of order  $r(0 < r < 1)$ . Weigh

$$\begin{aligned} \alpha^{(r)}X^{(0)} &= \alpha^{(1)}X^{(1-r)} \\ &= (\alpha^{(1)}x^{(1-r)}(1), \alpha^{(1)}x^{(1-r)}(2), \dots, \alpha^{(1)}x^{(1-r)}(m) \end{aligned} \tag{12}$$

is an accumulative sequence of order  $r(0 < r < 1)$ .

**Definition 4:** Set  $X^{(0)}$  and  $X^{(r)}$  as shown in Definition 2, then call

$$x^{(r)}(k + 1) = \beta_1x^{(r)}(k) + \beta_2 \tag{13}$$

is the discrete grey model of fractional accumulation operator, referred to as FDGM(1,1).

**Theorem 3:** The recursive solution of the FDGM(1,1) model is

$$\hat{x}^{(r)}(k + 1) = \beta_1^kx^{(0)}(1) + \frac{1 - \beta_1^k}{\beta_1}\beta_2 \tag{14}$$

or

$$\hat{x}^{(r)}(k + 1) = \beta_1^k(x^{(0)}(1) - \frac{\beta_2}{1 - \beta_1}) + \frac{\beta_2}{1 - \beta_1} \tag{15}$$

**Theorem 4:** Set  $X^{(0)}$  and  $X^{(r)}$  as shown in Definition 2, Then the parameter  $\hat{a} = [\beta_1, \beta_2]^T$  of FDGM(1,1) satisfies

$$\hat{a} = B^T(B^T B)^{-1}Y; \tag{16}$$

Where

$$B = \begin{bmatrix} X^{(r)}(1) & 1 \\ X^{(r)}(2) & 1 \\ \vdots & \vdots \\ X^{(r)}(m - 1) & 1 \end{bmatrix}, Y = \begin{bmatrix} X^{(r)}(2) \\ X^{(r)}(3) \\ \vdots \\ X^{(r)}(m) \end{bmatrix} \tag{17}$$

The proof process is the same as Theorem 2.

### 2.2.2 Parameter solving

From the perspective of the FDGM(1,1) model structure, if  $r$  is determined,  $\beta_1, \beta_2$  can be calculated by least squares. Taking into account the nonlinear characteristics of the model, the determination of parameters can be solved by a nonlinear optimization model with the smallest target error. This paper uses the average absolute percentage error to calculate.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}(i) - y(i)}{y(i)} \right| \times 100\% \tag{18}$$

The parameter solving process of FDGM(1,1) is as follows:

$$\min \quad MAPE \tag{19}$$

$$\begin{cases} 0 < r < 1 \\ [\alpha, \beta, \gamma]^T = (B^T B)^{-1} B^T Y; \\ \hat{x}^{(r)}(k+1) = \beta_1^k x^{(0)}(1) + \frac{1 - \beta_1^k}{\beta_1} \beta_2 \\ k = 2, 3, \dots, m \end{cases}$$

The above model can be calculated by some intelligent algorithms such as Gray Wolf Optimization Algorithm(GWO), Genetic Algorithm(GA), Particle Swarm Optimization(PSO), etc. Since the intelligent algorithm only needs to determine a parameter  $r$ , the stability of the parameter solution is relatively high. At the same time, in the empirical and numerical simulations of this paper, it is found that  $r$  calculated by the Gray wolf optimization algorithm, genetic algorithm, and particle swarm algorithm is almost equal. For this reason, this paper only gives the results obtained by the particle swarm algorithm. The flow chart of the PSO algorithm is as follows:

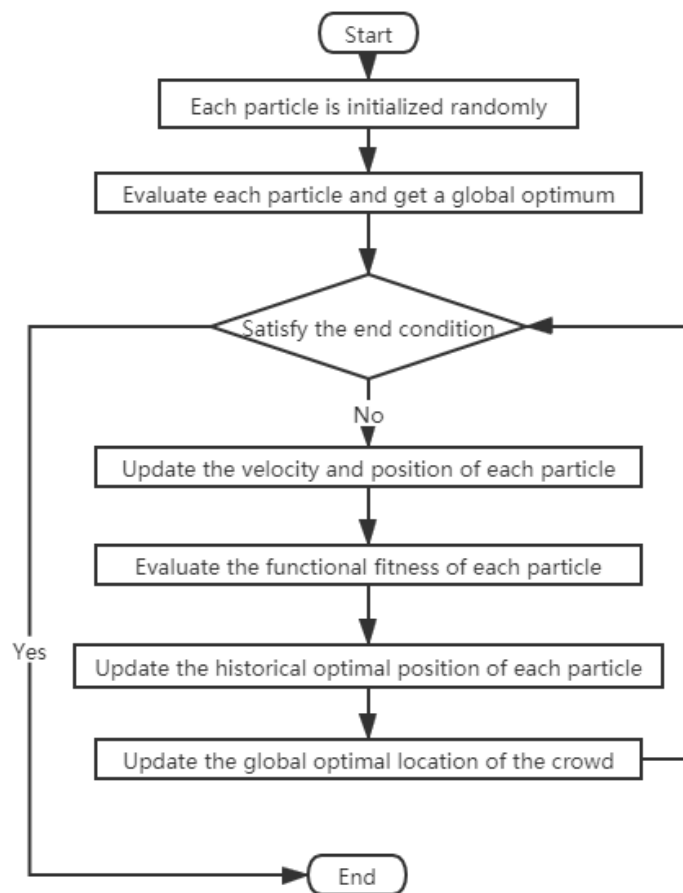


Fig. 1. PSO algorithm

### 3 Results and Discussion

In this paper, the annual gasoline consumption data in Chongqing (1997-2017) from the official website of the National Bureau of Statistics of China is used as the basic data, and the forecasting result is realized by Python programming. The annual gasoline consumption data in Chongqing with a data node of 21 is divided into two parts. The first part is the data set of the 1997-2013 time node as the training set of the model; the second part is the data set of the 2014-2017 time node, it is used to test the prediction performance of the trained model. In the training set, the 2011-2013 time node data set is used as the validation set when training the model, which is the target value when the PSO algorithm is used to search for the optimal parameters.

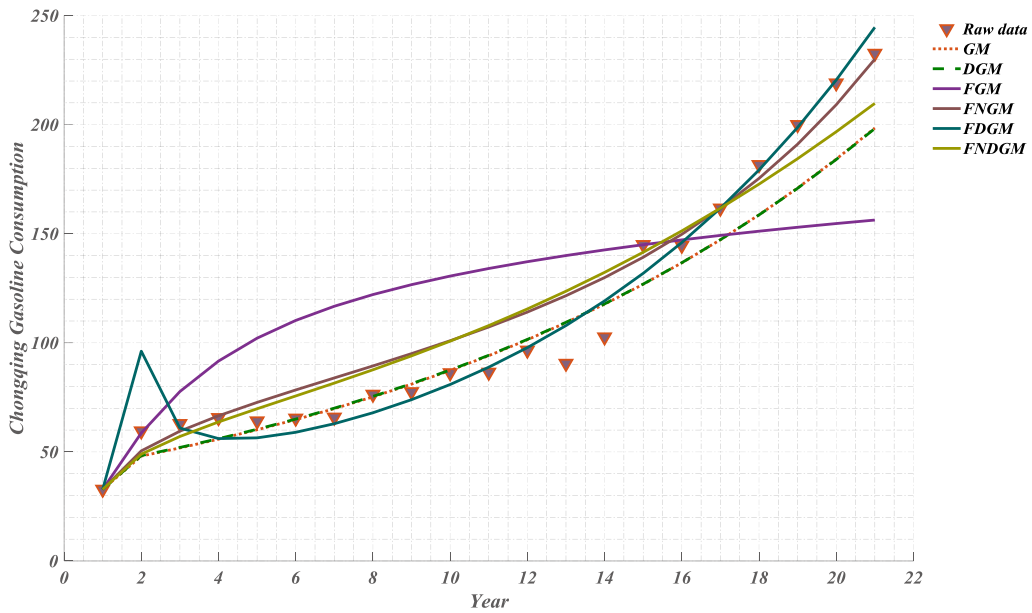


Fig. 2. Comparison of prediction results

Table 1. Predicted results of each model and MAPE value

Year	Raw data	GM	NGM	DGM	NDGM	FGM	FNGM	FDGM	FNDGM
1997	32.82	32.82	32.82	32.82	32.82	32.82	32.82	32.82	32.82
1998	59.55	48.08	68.04	48.31	60.59	58.67	50.47	96.17	48.99
1999	62.92	51.81	70.69	52.03	140.53	77.66	59.50	60.94	57.07
2000	65.66	55.82	73.99	56.05	304.36	91.63	66.55	56.10	63.72
2001	64.14	60.15	78.07	60.37	758.83	102.13	72.69	56.44	69.77
2002	65.41	64.80	83.12	65.03	2311.71	110.26	78.39	58.98	75.64
2003	65.87	69.83	89.38	70.04	8648.65	116.76	83.90	62.92	81.54
2004	76.37	75.23	97.14	75.45	39455.03	122.12	89.40	67.94	87.64
2005	77.53	81.06	106.76	81.27	218784.50	126.66	95.05	73.94	94.02
2006	86.22	87.34	118.67	87.54	1475966	130.59	100.95	80.89	100.75
2007	86.46	94.11	133.43	94.29	12140186	134.07	107.26	88.82	107.91
2008	96.59	101.40	151.72	101.56	1.22E+08	137.18	114.09	97.80	115.53
2009	90.54	109.25	174.37	109.39	1.5E+09	140.01	121.58	107.89	123.64
2010	102.63	117.72	202.45	117.83	2.26E+10	142.59	129.90	119.20	132.29
2011	144.97	126.83	237.23	126.92	4.18E+11	144.97	139.22	131.85	141.50
2012	144.63	136.66	280.32	136.71	9.47E+12	147.18	149.74	145.96	151.30
2013	161.7	147.25	333.72	147.26	2.64E+14	149.24	161.70	161.70	161.70
2014	181.64	158.65	399.88	158.62	9.01E+15	151.16	175.36	179.23	172.73
2015	199.98	170.94	481.85	170.85	3.79E+17	152.97	191.05	198.74	184.41
2016	219.05	184.19	583.41	184.03	1.96E+19	154.67	209.12	220.46	196.74
2017	232.65	198.45	709.25	198.22	1.24E+21	156.27	230.01	244.62	209.76
<b>MAPE of validation</b>		2.75%	30.11%	2.76%	258.44%E+17	4.88%	0.65%	<b>0.37%</b>	1.56%
<b>MAPE of all</b>		9.83%	67.80%	9.80%	258.44%E+17	35.57%	12.55%	<b>8.78%</b>	13.37%

In order to reflect the generalization ability of the model used in this paper, we compare the other three grey models and use first-order accumulation and fractional accumulation to predict the data set. For models containing fractional accumulation operators, we also use PSO to adjust Parameters: For models without parameters, directly fit the training set to get the prediction result. MAPE is used as a quantitative indicator to compare the predictive capabilities of different models. The predictive results of each model and the MAPE

value are shown in Table 1. Comparing the eight models, FDGM(1,1) has the best generalization ability and the best prediction performance on the test set or all prediction data, and the MAPE value on the entire time series is 8.78%. The generalization performance shown on the test set is stronger. The MAPE value reaches 0.37%, which is much smaller than the other seven models, indicating that the discrete grey model under fractional order accumulation shows excellent predictive performance in the application of gasoline consumption. Forecasts after the year are more effective.

In order to more intuitively reflect the predictive ability of the model, this paper shows the prediction results of the six models with smaller overall MAPE values, as shown in Fig. 2. It can be clearly shown from the figure that the FDGM(1,1) forecast results are closer to the original time series, and are almost completely consistent in the forecast after 2012. Therefore, the grey discrete model of fractional accumulation operator optimized based on PSO algorithm presents excellent generalization performance in the forecast of gasoline consumption in Chongqing, and can be used as an effective tool for energy forecasting.

## 4 Conclusion

Based on the discrete grey theory of fractional accumulator optimized by PSO algorithm, this paper establishes a forecasting model of gasoline consumption in Chongqing. On the basis of the fractional discrete grey model, the PSO optimization algorithm is used to calculate the optimal fractional parameters. Judging from the forecast results of gasoline consumption in Chongqing, the fractional discrete grey model optimized by the PSO algorithm is significantly better than the other seven grey models, with higher prediction accuracy and higher reliability, and can be used as a quantitative prediction method for energy consumption Application in practice.

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## Competing Interests

Authors have declared that no competing interests exist.

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